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Risk Parity

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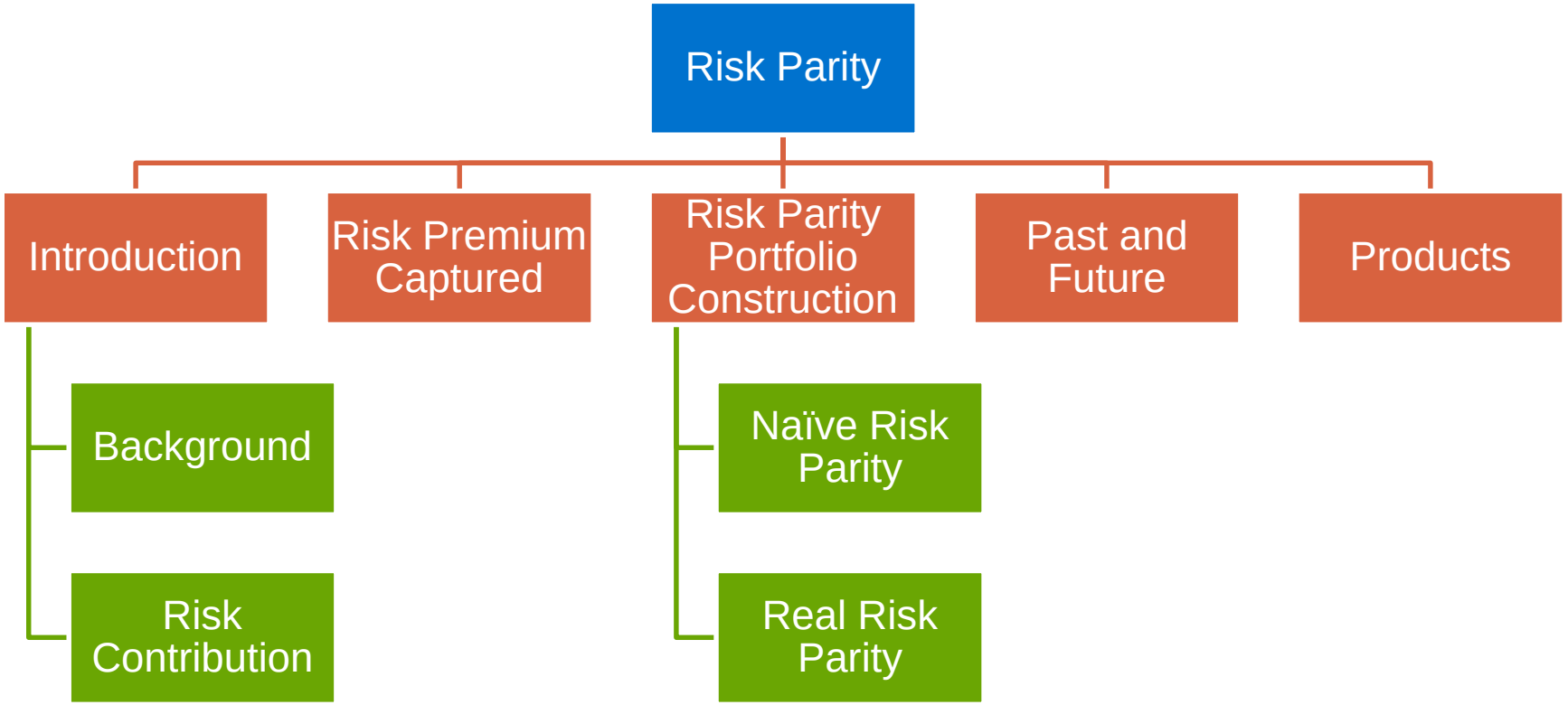
Outline


Risk Parity
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Risk Contribution
Naïve Risk Parity
Real Risk Parity

Risk Parity: Background

- What is Risk Parity?
  - A form of asset allocation
  - Instead of equalizing capital, it aims at equalizing Risk Contribution
  - It has been one of the trendiest strategy since the financial crisis
  - Concept coined by Edward Qian (Panagora Asset Management)
  - First Risk Parity fund: Bridgewater Associate 'All Weather' fund

- Why Risk Parity?
  - Strong historical performance capturing well-known risk premiums
  - Does not reply on expected returns (no active views)
  - Does not rely on optimization
  - Mean-variance optimal if each asset has the same Sharpe ratio

Risk Contribution (Two Assets)

$$w_E = 0.6, \quad w_B = 0.4, \quad \sigma_E = 0.15, \quad \sigma_B = 0.05, \quad \rho = 0.2$$

$$\sigma_p^2 = w_E^2\sigma_E^2 + w_B^2\sigma_B^2 + 2\rho w_E w_B \sigma_E \sigma_B = (9.6\%)^2$$

$$\textit{Risk Contribution of Stock} = \frac{w_E^2\sigma_E^2 + \rho w_E w_B \sigma_E \sigma_B}{\sigma_p^2} = 92\%$$

$$\textit{Risk Contribution of Bond} = \frac{w_B^2\sigma_B^2 + \rho w_E w_B \sigma_E \sigma_B}{\sigma_p^2} = 8\%$$

Risk Contribution (General)

$$Z = \sum w_j R_j \qquad \mathrm{S}_{ij} = Cov(R_i, R_j)$$

$$Risk\ Contribution\ of\ Asset\ i\ = \frac{w_i (\mathrm{S}w)_i}{w^T \mathrm{S} w}$$

Return Contribution vs. Risk Contribution

$$\max: U(w) = \max_{W}: w^T m - \frac{l}{2} w^T \Sigma w$$

- For mean-variance optimal portfolio:

$$FOC: m = l \times \Sigma w$$

$$\text{Return Contribution of Asset } i = \frac{w_i m_i}{w^T m}$$

$$= \frac{w_i (\Sigma w)_i}{w^T \Sigma w} = \text{Risk Contribution of Asset } i$$

Loss Contribution: Example

Portfolio: $w_{stock} = 0.6$, $w_{bond} = 0.4$

When the portfolio hits −5% return ($VaR_{portfolio} = -5\%$):

- Conditional expected return from stock = $CVaR_{stock} = -4.5\%$
  - stock contributes 90% loss
- Conditional expected return from bond = $CVaR_{bond} = -0.5\%$
  - bond contributes 10% loss

Loss Contribution: Evidence (1926 ~2004)

TABLE 1.2 Monthly Return Statistics of Indices and the 60/40 Portfolio

	S&P 500	US LT Government	60/40 Portfolio
Average return	0.98%	0.46%	0.78%
Standard deviation	5.61%	2.27%	3.61%
Correlation to S&P 500	1.00	0.14	0.97

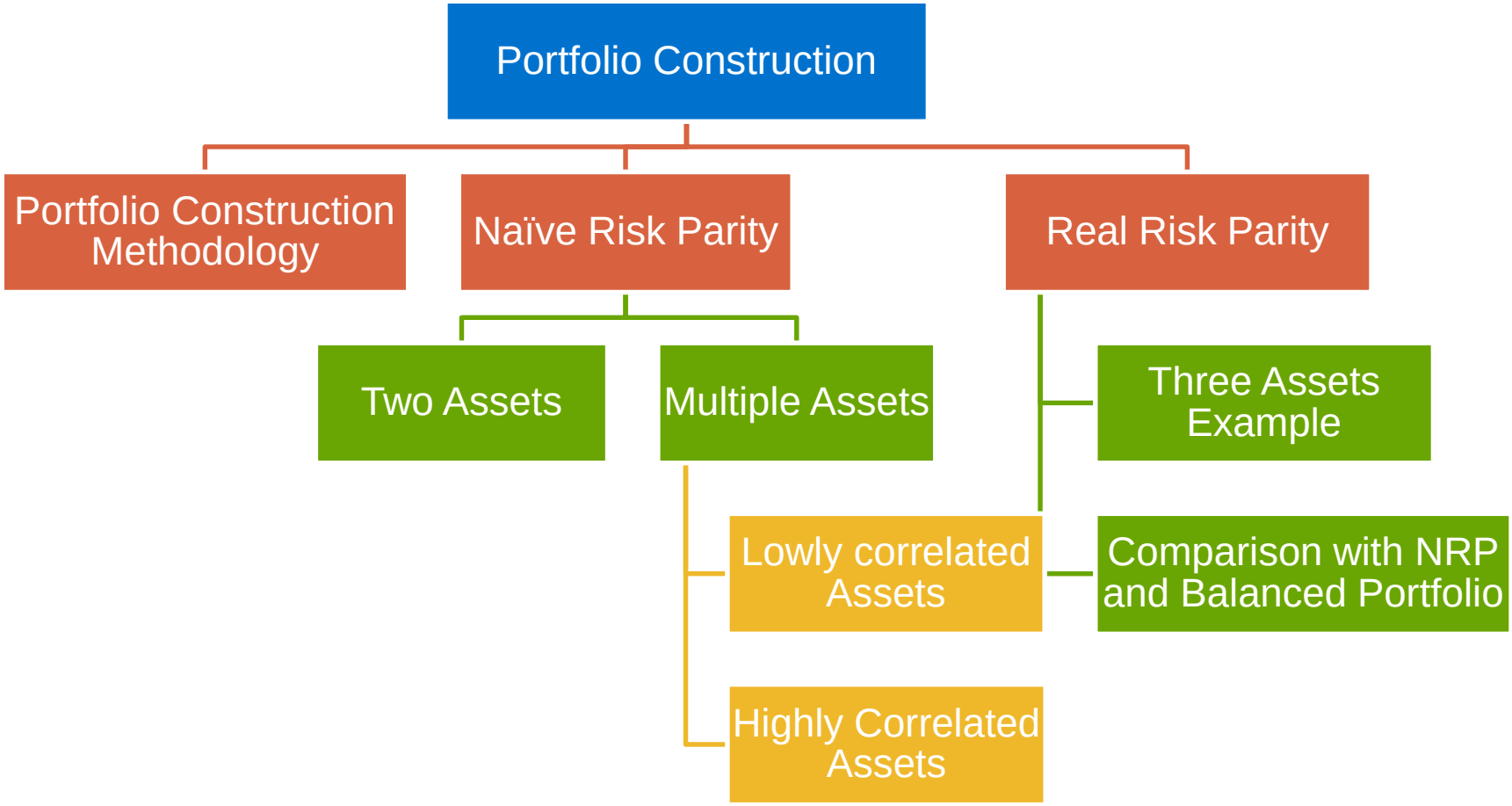
TABLE 1.3 Realized Loss Contributions from Stocks of the 60/40 Stock/Bond Portfolio

Loss (%)	Loss Contribution (%)	Number of Months
−4 to −3	89.8	45
−5 to −4	92.7	23
−6 to −5	88.1	11
−7 to −6	99.5	9
−8 to −7	90.1	8
Below −8	102.4	12

Risk Premiums Captured

- Equal risk exposure to different risk premiums vs Equal risk exposure to different risky assets
- Three primary risk premiums
  - Equity risk premium
    - Compensation for providing capital to overall economy
  - Interest rate risk premium
    - Compensation for lending money over long term. Upward slope of term-structure. Positive term spread.
  - Inflation risk premium
    - Inflation-linked bonds, commodities (no dividend). Mainly used as a hedge against high inflation.
  - * Dynamic Factor risk premium
    - Carry trade: buying high-yielding currencies and sell low-yielding currencies

Risk-Parity Portfolio Construction



Risk Parity Methodology

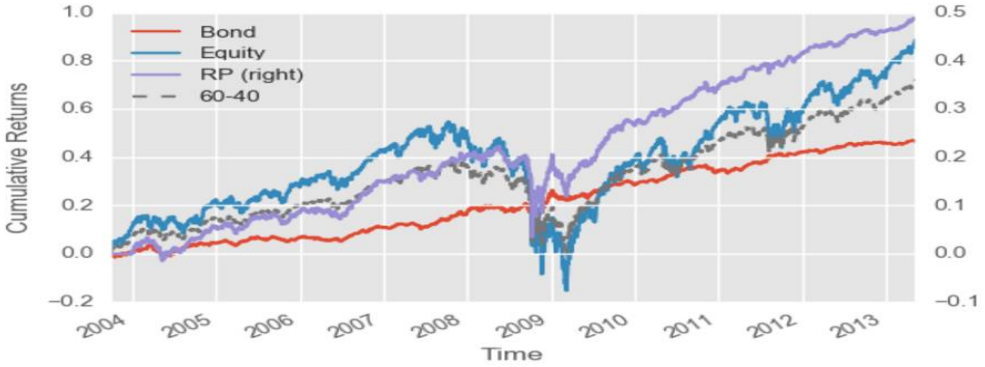
- Methodology
  - **Naive (correlation blind) Risk Parity**
    - Weight is inversely proportional to volatility forecast: $w_i \propto \frac{1}{\sigma_i^2}, i = 1 \sim N$
    - Asset with the same volatility forecast will be weighted equally
  - **Real Risk Parity**
    - Equalization of Risk Contribution: $RC_i = \frac{w_i(\boldsymbol{\Sigma w})_i}{\boldsymbol{w^T \Sigma w}} = \frac{\boldsymbol{1}}{\boldsymbol{N}}$
    - Asset with the largest volatility and average correlation has the smallest weight
  - ***Cluster Risk Parity**
    - Weight assets within each cluster C_j using NRP: $\mathrm{v_i} \propto \frac{1}{\sigma_i^2}$
    - Weight cluster portfolios using NRP: $\mathrm{w}_{C_j} \propto \frac{1}{\sigma_{C_j}^2}$
    - $w_i = v_i \times w_{C_j}$ for all asset belong to cluster C_j

NRP- Two Lowly Correlated Assets

Investment Universe

Asset Class	Ticker	Name
Equity	SPY	SPDR S&P 500 ETF Trust
Bond	AGG	iShares Barclays Aggregate Bond Fund

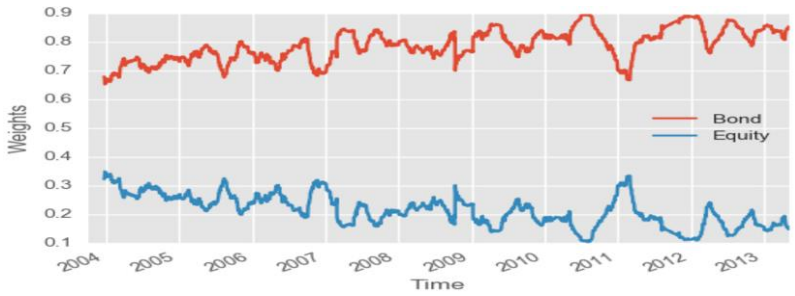
Cumulative Performance



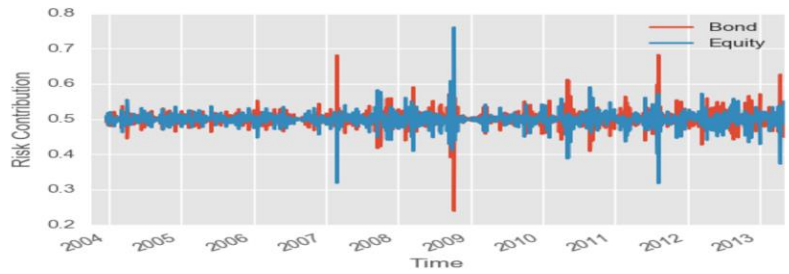
Portfolio Performance Analysis

	Ann_Ret	Ann_Vol	Sharpe	MaxDD	DD_Start	DD_End	Var(95)	ES(95)	TR(95)
Bond	4.835120	5.395134	0.869057	-13.419330	2008-09-09	2008-10-10	6.631598	11.187464	10.083290
Equity	9.200897	20.626394	0.343045	-70.003588	2007-10-09	2009-03-09	29.992710	50.428582	23.278046
60-40	7.454587	12.336246	0.542791	-38.820582	2007-10-09	2009-03-09	17.056954	29.750930	14.138996
RP	5.058949	5.844693	0.836415	-18.851249	2008-05-19	2008-10-10	6.798142	12.334120	9.911632

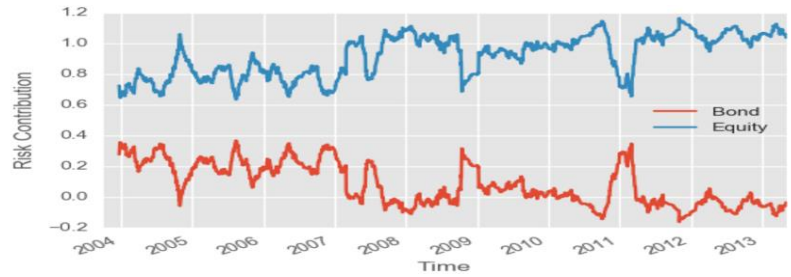
Asset Weight of NRP Portfolio



Risk Contribution of NRP Portfolio



Risk Contribution of Traditional 60-40 Portfolio



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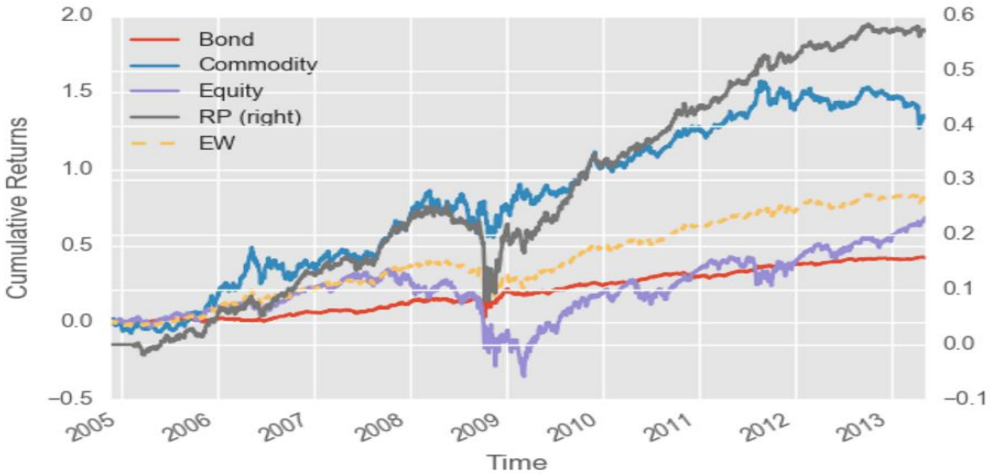
NRP- Multiple Lowly Correlated Assets

Investment Universe

Asset Class	Ticker	Name
Equity	SPY	SPDR S&P 500 ETF Trust
Bond	AGG	iShares Barclays Aggregate Bond Fund
Commodity	GLD	SPDR Gold Trust (ETF)

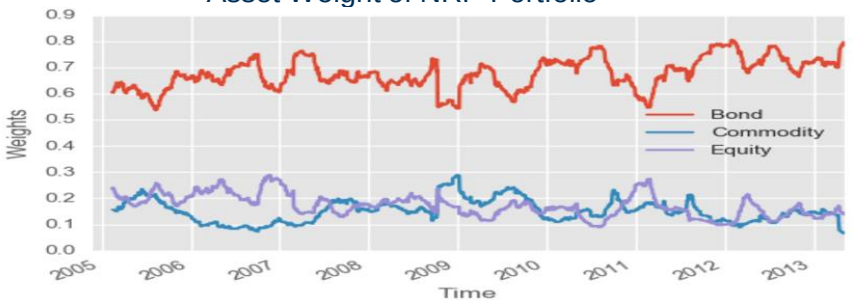
Cumulative Performance



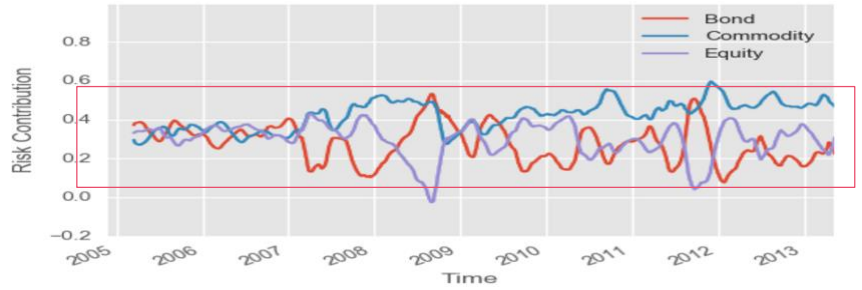
Portfolio Performance Analysis

	Ann_Ret	Ann_Vol	Sharpe	MaxDD	DD_Start	DD_End	Var(95)	ES(95)	TR(95)
Bond	4.996839	5.499607	0.880895	-13.419330	2008-09-09	2008-10-10	6.620586	11.458563	10.658624
Commodity	15.818521	20.766981	0.657660	-30.882152	2011-08-22	2013-04-15	33.182029	49.716340	18.662359
Equity	8.099882	21.581604	0.267522	-70.003588	2007-10-09	2009-03-09	32.246978	52.991024	23.579010
EW	9.638414	10.467623	0.868442	-25.176050	2008-05-20	2008-11-20	15.612854	24.256707	10.977665
RP	6.767600	6.289351	1.044563	-17.630160	2008-05-20	2008-10-10	8.385743	13.597792	9.771122

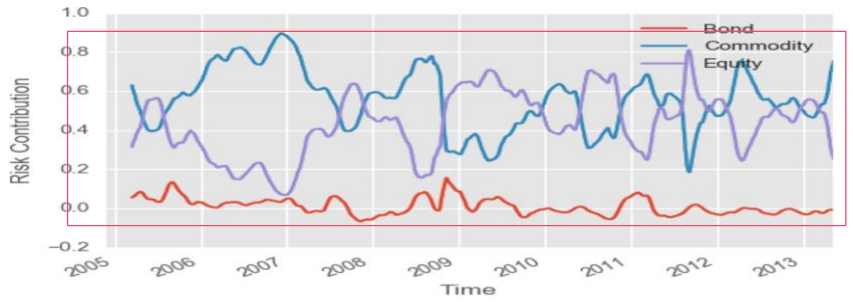
Asset Weight of NRP Portfolio



Risk Contribution of NRP Portfolio



Risk Contribution of Equally Weighted Portfolio



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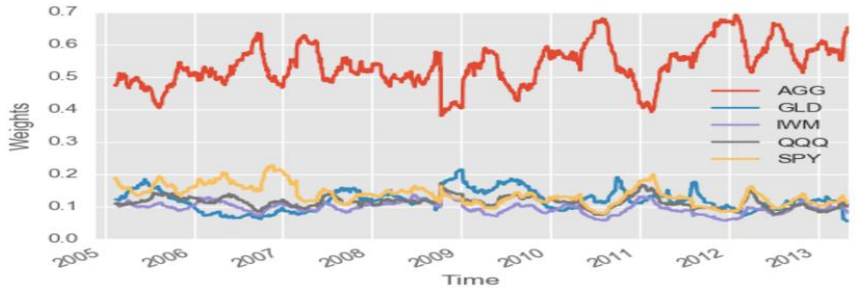
NRP- Multiple Highly Correlated Assets

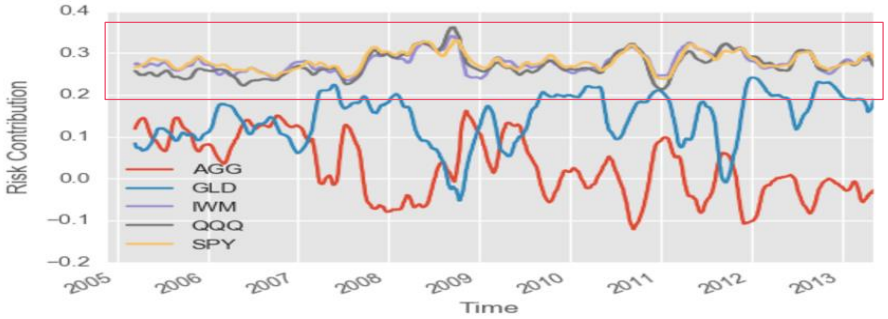
Investment Universe

Asset Class	Ticker	Name
Equity	SPY	SPDR S&P 500 ETF Trust
Equity	QQQ	PowerShares QQQ ETF Trust
Equity	IWM	iShares Russell 2000 Index (ETF)
Bond	AGG	iShares Barclays Aggregate Bond Fund
Commodity	GLD	SPDR Gold Trust (ETF)

Asset Weight of NRP Portfolio

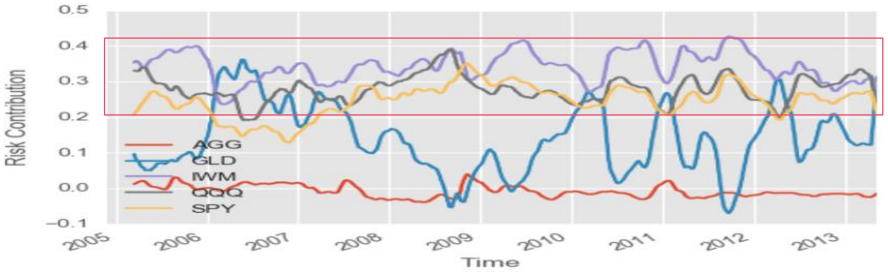


Risk Contribution of NRP Portfolio



Cumulative Performance

Risk Contribution of Equally Weighted Portfolio



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RRP- Three Asset

- Risk attributed to correlation account for a large part especially when correlations are high.
- Most times RP portfolios are levered, thus small difference in asset allocation will be amplified.

Asset allocation

	AGG	GLD	SPY
NRP	0.68	0.15	0.17
RRP	0.71	0.13	0.16

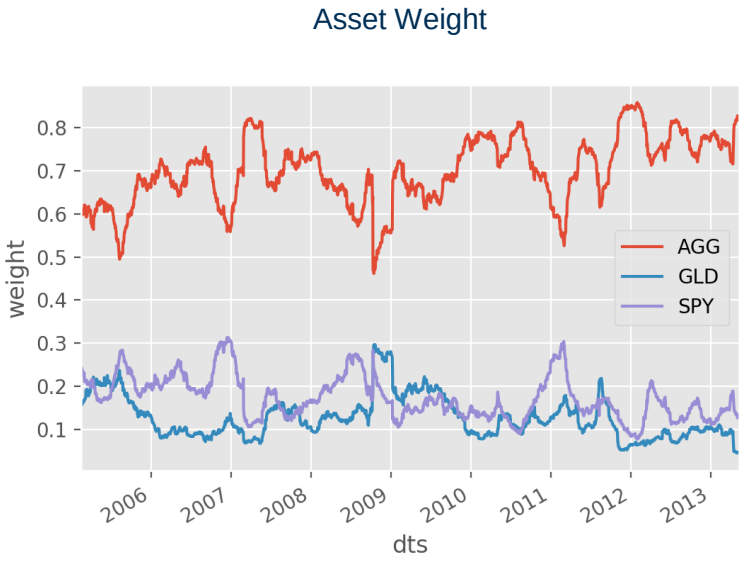
Correlation for the whole period

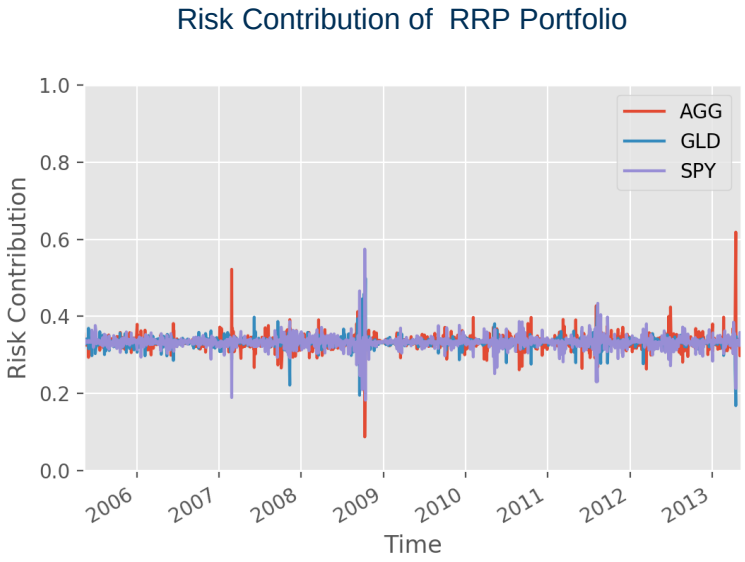
	AGG	GLD	SPY
AGG	1	0.08	-0.11
GLD	0.08	1	0.08
SPY	-0.11	0.08	1

Correlation for during financial crisis

	AGG	GLD	SPY
AGG	1	0.11	-0.02
GLD	0.11	1	-0.03
SPY	-0.02	-0.03	1

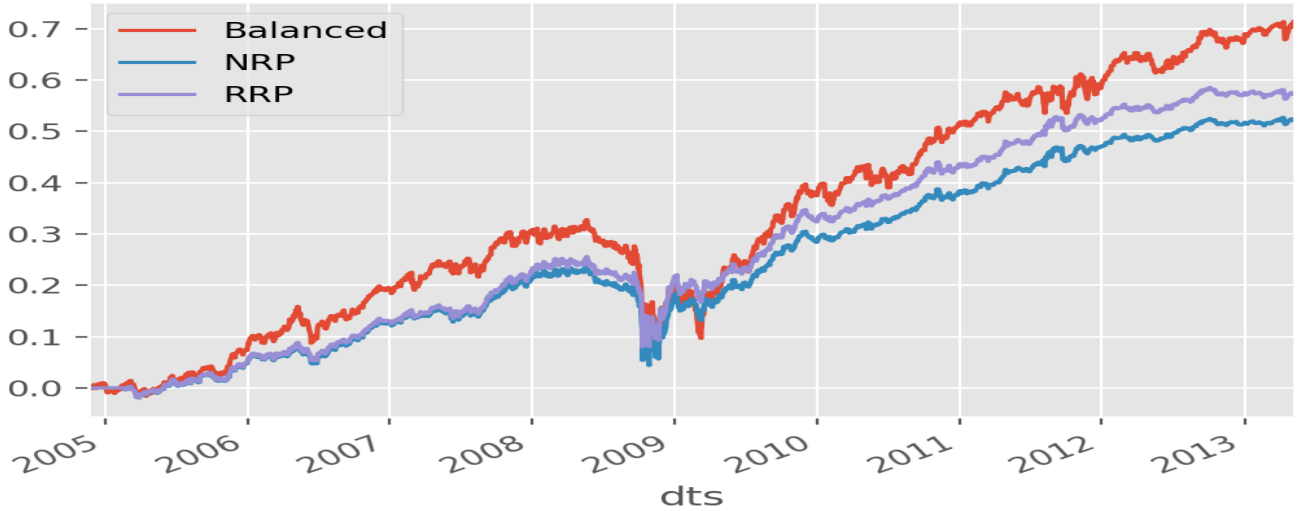
RRP- compare with NRP and balanced portfolio





RRP- compare with NRP and balanced portfolio

	Balanced	NRP	RRP
Sharpe	0.842	0.981	1.07



Cumulative return comparison

1994 Crash of Bond Market History Lessons

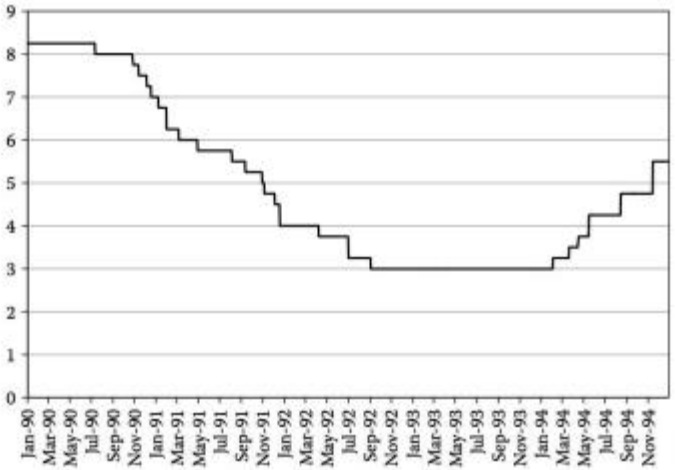


FIGURE 6.1 The Fed fund target rate from 1990 to 1994.

FIGURE 6.3 US 10-year treasury bond yield from 1990 to 1994.

TABLE 6.2 Excess Return, Volatility, and Sharpe Ratio of Citigroup UST Indices

	1–3 years	3–7 years	7–10 years	20 years+	Index
Excess return	−3.6%	−7.0%	−9.6%	−12.0%	−7.3%
Risk	1.6%	3.6%	5.8%	8.6%	4.2%
Sharpe	−2.28	−1.96	−1.66	−1.38	−1.75

TABLE 6.4 Absolute Returns of a 60/40 Portfolio and a Static Risk Parity Portfolio

	60/40 (%)	Risk Parity (%)
Return 1994	−2.7	−5.9
Return 2008	−22.9	−14.0

Historical performance in US and Japan

TABLE 6.8 Simulated Returns for Risk Parity Portfolios and 60/40 Portfolio

	CPI (%)	Risk Parity Foundation (%)	Risk Parity Dynamic (%)	60/40 (%)
1970–1979	7.4	10.5	11.7	6.5
1980–1989	5.1	14.3	14.9	15.7
1990–1999	2.9	12.2	13.5	13.9
2000–2009	2.5	4.3	5.2	0.4

TABLE 6.7 Annualized Percentage Change in CPI and Annualized Returns of Financial Assets

	CPI (%)	Cash (%)	GSCI (%)	TIPS (%)	US Bond (%)	S&P 500 (%)
1970–1979	7.4	6.5	21.2	10.4	6.7	5.9
1980–1989	5.1	9.1	10.7	6.6	12.1	17.6
1990–1999	2.9	5.0	3.9	5.3	7.2	18.2
2000–2009	2.5	2.8	5.1	5.7	5.3	−3.5

TABLE 6.6 Return and Risk of 60/40 and Risk Parity Japan Portfolios

	60/40 Japan (%)	Risk Parity Japan (%)
Return	0.98	6.83
Risk	10.62	8.23

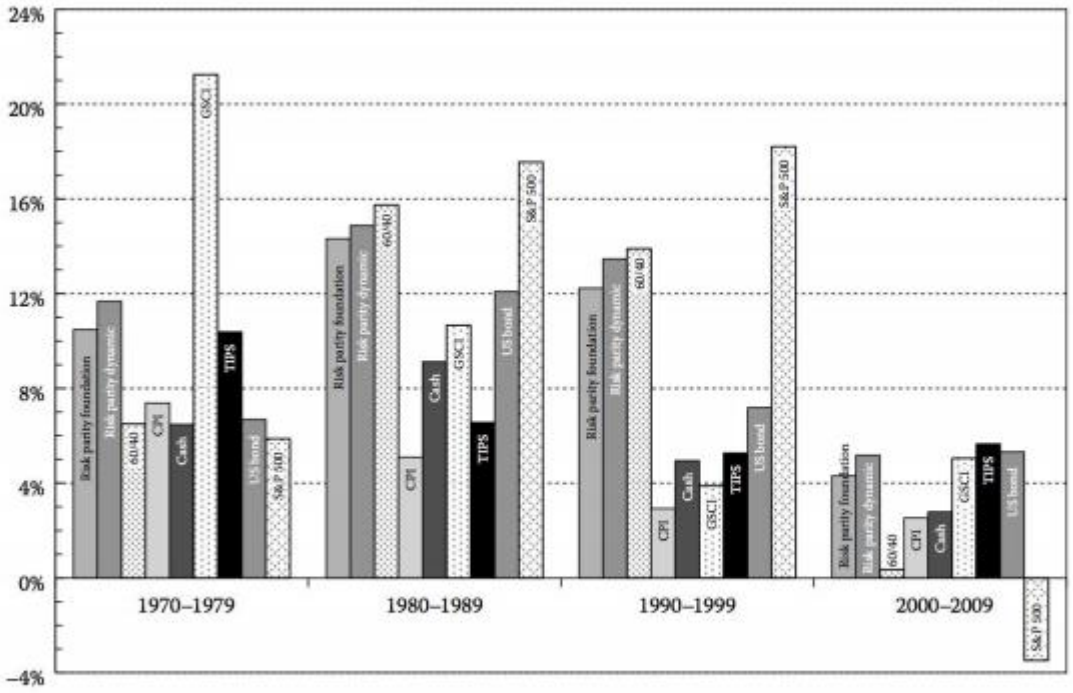


FIGURE 6.22 Returns of underlying assets and risk parity portfolios.

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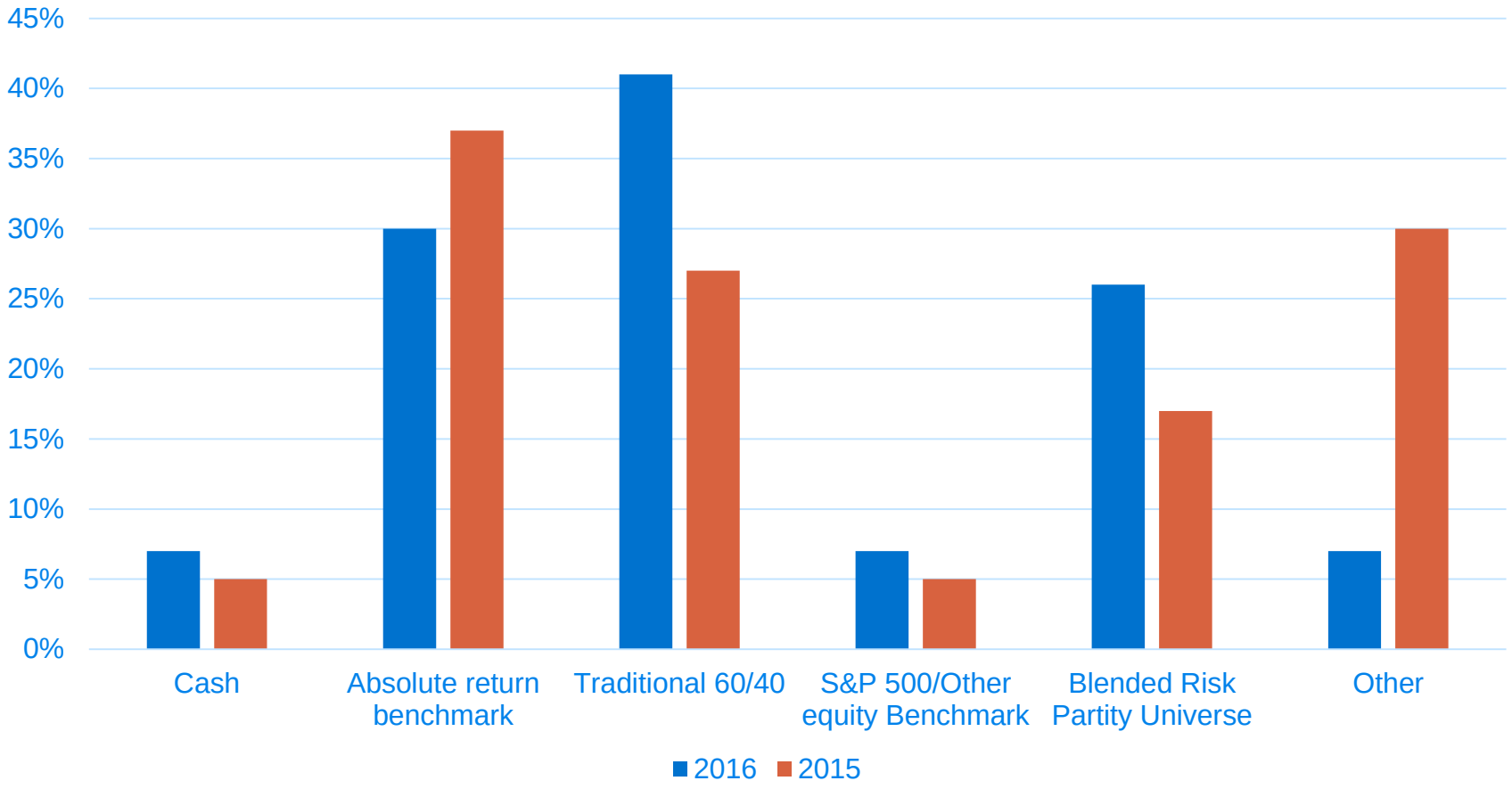
Risk Parity Funds

The Big Names in Risk Parity

	Total Assets (at end of 2014)	2014 Return	2015 Return	2016 Return
Bridgewater All Weather Strategy	$82 billion	8.6%	-7.00%	10.00%
AQR Global Risk Premium Fund	$30 billion	9.4%	-7.20%	11.80%
Invesco Balanced-Risk Allocation Fund	$19.5 billion	5.5%	-4.68%	10.97%
Salient Risk Parity Fund	$2.48 billion	13.58%	-11.58%	14.13%
Putnam Dynamic Risk Allocation	$1.3 billion	2.69%	-6.32%	8.74%
Columbia Adaptive Risk Allocation Fund	$243 million	10.22%	-4.85%	9.27%

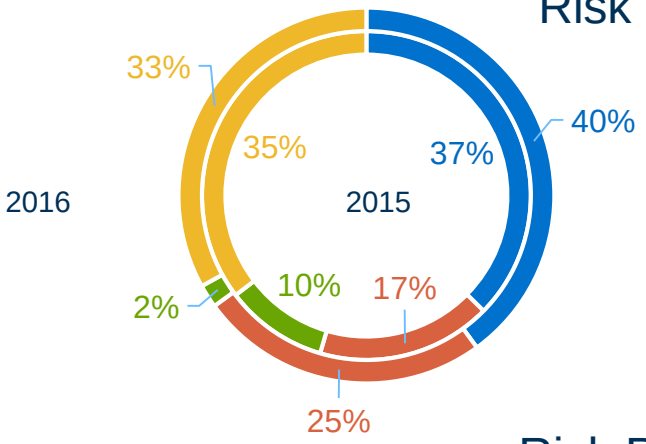
Source: Morningstar, Forbes, Invesco, AQR, Columbia, Putnam, Salient

How is it benchmarked



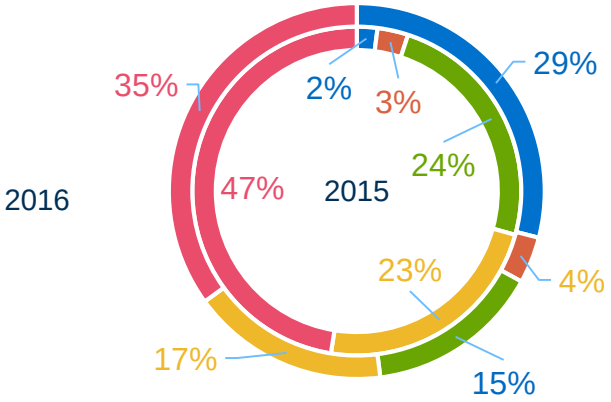
Who is the target audience?

Risk Parity users by asset owner type



- Corporate pension funds
- Public pension/sovereign wealth
- Endowment/foundation
- Other

Risk Parity users by asset size



- Less than $500 million
- $500 million - $1 billion
- $1 billion - $5 billion
- $5 billion - $15 billion
- More than $15 billion

Future Risks

How concerned are you about the following issues, if at all?

	NOT AT ALL	A LITTLE	MODERATELY	QUITE	EXTREMELY
The use of leverage/ counterparty risk	22%	33%	33%	11%	0%
Performance	15%	22%	37%	22%	4%
Peer risk	38%	21%	38%	0%	4%
Assets without a risk premium/prohibited by IPS	37%	33%	22%	7%	0%
Board and staff education	19%	35%	15%	23%	8%
No explicit bucket to put it in	62%	27%	4%	8%	0%
Not enough viable manager offerings	65%	12%	15%	8%	0%
The passive approach some vendors take	62%	19%	8%	8%	4%
Dynamic asset allocation policy is preferable	35%	42%	15%	4%	4%
Timing	31%	27%	19%	15%	8%
Transparency	19%	27%	35%	15%	4%
How to benchmark	24%	40%	12%	16%	8%

Non-users: What concerns have led you to not allocate or not consider an allocation towards risk parity strategies?

	2016	2015
The use of leverage/counterparty risk	33%	42%
Performance	33%	34%
Board and staff education	27%	20%
Timing (i.e. implementing a portfolio today given the valuation of bonds)	20%	6%
Benchmarking	20%	4%
Not enough viable manager offerings	13%	15%
Peer risk	7%	22%
Assets without a risk premium/prohibited by IPS	7%	22%
Transparency	7%	7%
No explicit bucket to put it in	0%	19%
The passive approach some vendors take	0%	15%
Dynamic asset allocation policy is preferable	0%	12%
Other	13%	26%

Appendix: Loss Contribution: Definition

- What's the contribution of asset *i* to the total loss of the portfolio ?

$$CVaR_i = E(w_i R_i \mid Z = VaR) \quad \text{where} \quad \sum_j CVaR_j = VaR$$

$$Loss\ Contribution\ of\ asset\ i = \frac{CVaR_i}{VaR}$$

Appendix: Loss Contribution vs. Risk contribution

- Under assumption of normally distributed returns:

$$VaR = k\sqrt{w^T \Sigma w} \quad \text{where } k = 2.33 \text{ for } 99\% \text{ quantile}$$

$$CVaR_i = w_i \frac{\partial VaR}{\partial w_i} = k \times w_i \frac{(\Sigma w)_i}{\sqrt{w^T \Sigma w}}$$

$$\text{Loss Contribution of Asset } i = \frac{CVaR_i}{VaR}$$

$$= \frac{w_i (\Sigma w)_i}{w^T \Sigma w} = \text{Risk Contribution of asset } i$$

Appendix: Risk Parity Portfolio:

- A Reverse Problem (Equal Risk Contribution):

$$\text{Solve for } w_i: \quad \frac{w_i (Sw)_i}{w^T S w} = \frac{1}{N} \quad \text{for} \quad i = 1 \sim N$$

- Uncorrelated Returns:

$$\text{Solutions } w_i \; \mu \; \frac{1}{s_i^2} \quad \text{for} \quad i = 1 \sim N$$

Equal Return Contribution = Equal Risk Contribution = Equal Loss Contribution