Tail Risk Mitigation using Hidden Market Model

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2016-04-19

Introduction: Regime Switching Models

A regime in financial time series can be defined as a continuous period of time when certain specific characteristic or dynamics are observed. When such dynamics change, a new regime is said to be observed. Regimes can therefore be defined by various aspects:

Based on the return characteristics, a market can be segmented into bull (bullish) and bear (bearish) regimes. If such switching (turning) point can be predicted, trading strategies can be enhanced to act on this insight.

Regime can also be characterized by realized or implied volatilities. A good prediction of such regime switching point will be beneficial for developing regime-specific risk models or trading strategies.

Furthermore, regime can be defined by risk appetite or systemic risk in the market. A market associated with high systemic risk is dangerous and fragile, and traders and investors should be cautious not to take excessive risk during these periods.

Tail Risk Mitigation using Hidden Market Model

One application of Hidden Markov Model for regime switch detection is highlighted in a paper published by Kent Daniel (Daniel, 2012). In that paper, he claimed that momentum strategies have historically provided excessive returns with little systematic risk. However, these momentum portfolios, such as a US-equity momentum portfolio during 1920-2010 testing period, suffered infrequent but severe drawdowns. He argues that by utilizing a Hidden Markov model with two latent (hidden) states, 'turbulent' and 'calm', that avoids taking momentum positions during the turbulent states, can prevent most of the high-loss episodes. In summary, this strategy is a conditional momentum strategy:

- When the ex-ante probability that the current latent state is turbulent exceeds 70%, unwind all momentum positions and switch to holdings of risk-free assets (such as treasuries)
- Otherwise a momentum strategy is employed to establish positions on recent market winners

Now, based on Kent's paper, I will illustrate the model specification and the mathematical derivation. The parameters of this Hidden Markov Model for Momentum portfolio (HMM-Mom) is estimated numerically using *Maximum likelihood Estimation*. Our goal is to maximize the total log likelihood L subject to parameters set θ:

$$\theta_{ML} = \arg\max_{\vartheta \in \theta} L(\vartheta)$$

$$\theta = \left\{ \begin{matrix} \alpha(C), \beta^0(C), \beta^U(C), \sigma_{mom}(C) \\ \alpha(T), \beta^0(T), \beta^U(T), \sigma_{mom}(T) \\ \mu(C), \sigma_M(C), \mu(T), \sigma_M(T) \\ \Pr(S_t = C | S_{t-1} = C), \Pr(S_t = T | S_{t-1} = T) \end{matrix} \right\}$$

Where L is the summation of log likelihood of observing y_t based on the information set available at time $t-1$ for each period from 1 to T:

$$L = \sum_{t=1}^{T} \log(\Pr(y_t | F_{t-1}))$$

Here, y_t is the vector of observable variables at time t:

$$y_t = (I_t^U,\ R_t^M,\ R_t^{mom})$$

F_{t-1} denotes the information set at time $t-1$:

$$F_{t-1} = \{y_{t-1}, y_{t-2}, \dots, y_1\}$$

In order to compute the above likelihood, for each period, we need to know the likelihood of observing y_t given information set F_{t-1} at time $t-1$. To achieve this, we decompose $\Pr(y_t|F_{t-1})$ into two terms M and N:

$$\Pr(y_t|F_{t-1}) = \sum_{S_t} \Pr(y_t, S_t = s_t | F_{t-1}) = \sum_{S_t} \Pr(y_t | S_t = s_t, F_{t-1}) \Pr(S_t = s_t | F_{t-1})$$

$$= \sum_{S_t} M \times N$$

The first term, M, can be estimated by a joint Gaussian distribution. Specifically, we model the return generating process for momentum portfolio and the market portfolio (net of risk-free rate) as:

$$R_t^{mom} = \alpha(S_t) + (\beta^0(S_t) + \beta^U(S_t) \cdot I_t^U)\ R_t^M + \sigma_{mom}(S_t)\varepsilon_t^{mom}$$

$$R_t^M = \mu(S_t) + \sigma_M(S_t)\varepsilon_t^M$$

Together, they form a joint Gaussian distribution:

$$M = \Pr(y_t | S_t = s_t, F_{t-1})$$

$$= \frac{1}{\sigma_{mom}(s_t)\sqrt{2\pi}}\ exp\left\{-\frac{\left(\varepsilon_t^{mom^2}\right)}{2}\right\} \times \frac{1}{\sigma_M(s_t)\sqrt{2\pi}}\ exp\left\{-\frac{\left(\varepsilon_t^{M^2}\right)}{2}\right\}$$

Where

$$\varepsilon_t^{mom}(s_t) = \frac{1}{\sigma_{mom}(s_t)} \left(R_t^{mom} - \alpha(S_t) - \left(\beta^0(S_t) + \beta^U(S_t) \cdot I_t^U\right) R_t^M\right)$$

$$\varepsilon_t^M(s_t) = \frac{1}{\sigma_M(s_t)} \left(R_t^M - \mu(s_t)\right)$$

The second term, N, is the probability of the hidden state at time t to be in s_t given the observed information set F_{t-1} at time $t-1$:

$$N = \Pr(S_t = s_t | F_{t-1}) = \sum_{S_{t-1}} \Pr(S_t = s_t, S_{t-1} = s_{t-1} | F_{t-1})$$

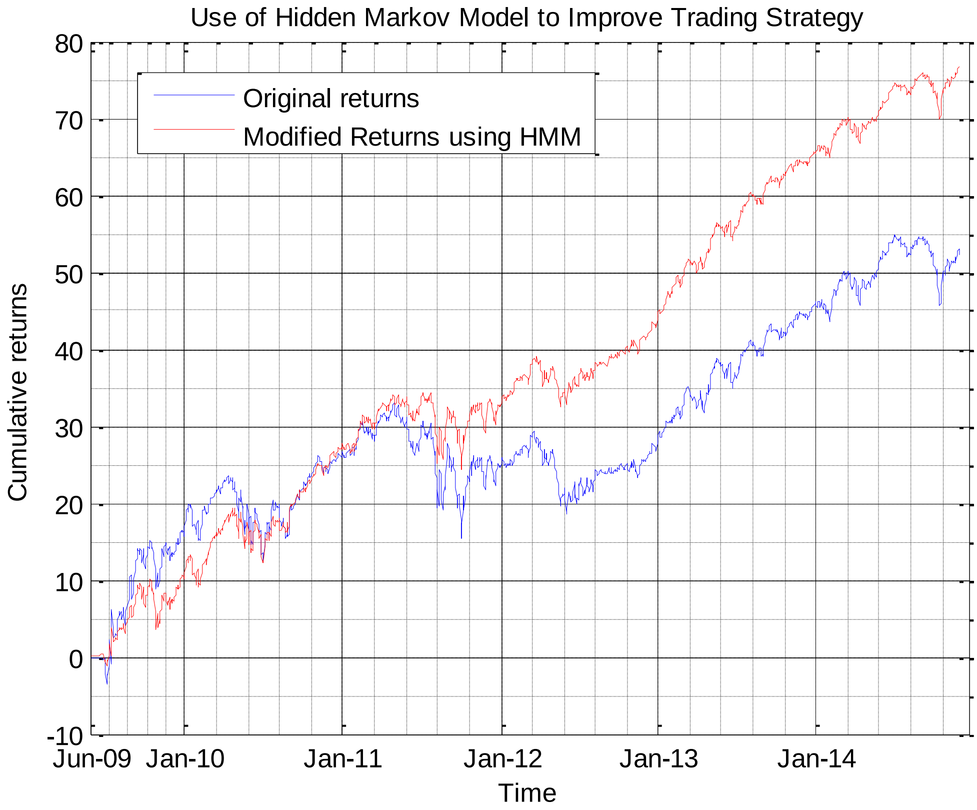
$$= \sum_{S_{t-1}} \Pr(S_t = s_t | S_{t-1} = s_{t-1}, F_{t-1}) \Pr(S_{t-1} = s_{t-1} | F_{t-1})$$

$$= \sum_{S_{t-1}} \Pr(S_t = s_t | S_{t-1} = s_{t-1}) \Pr(S_{t-1} = s_{t-1} | F_{t-1})$$

Where $\Pr(S_t = s_t | S_{t-1} = s_{t-1})$ is determined by the transition matrix. $\Pr(S_{t-1} = s_{t-1} | F_{t-1})$ is assumed to be known at the current time t. Specifically, at time $t = 1, \Pr(S_{t-1} = s_{t-1} | F_{t-1}) = \Pr(S_0 = s_0)$, is initially the unconditional stationary probability, which is determined by the transition matrix that we want to estimate; at $t > 1$, we update state probability $\Pr(S_t = s_t | F_t)$ for each period according to the Bayes' rule:

$$\Pr(S_t = s_t | F_t) = \Pr\left(S_t = s_t \middle| F_{t-1,} y_t\right) = \frac{\Pr(S_t = s_t, y_t | F_{t-1})}{\Pr(y_t | F_{t-1})}$$

After incorporating this aforementioned Hidden Markov Model, an original equity momentum strategy is substantially enhanced to deliver less risk and higher expected returns. This is achieved by merely reducing the exposure during turbulent states suggested by the HMM based regime switching model:


Use of Hidden Markov Model to Improve Trading Strategy
Original returns
Modified Returns using HMM
Cumulative returns
80
70
60
50
40
30
20
10
0
-10
Jun-09
Jan-10
Jan-11
Jan-12
Jan-13
Jan-14
Time