
TEST FOR RANDOM WALK HYPOTHESIS

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DISCRETE-TIME RANDOM WALK

Random walk process is also called Brownian Motion Process. The simplest form of random walk is modeled as a process where the current value is equal to the past value plus a white noise with zero mean and constant variance. Algebraically, this process can be denoted as:

$$y_t = y_{t-1} + \varepsilon_t$$

Therefore, it has no memory, and the best prediction for the next future value is just the current realization. The change term $\Delta y_t = y_t - y_{t-1}$ is absolutely random and unpredictable.

In a more general setting, a random walk can also show a deterministic trend term, and the process can therefore be denoted as:

$$y_t = \alpha + y_{t-1} + \varepsilon_t$$

This process shows both a deterministic trend and a stochastic trend. It can be rewritten as:

$$y_t = y_0 + at + \sum_{k=1}^t \varepsilon_k$$

Where $y_0 + at$ the deterministic trend is portion and $\sum_{k=1}^t \varepsilon_k$ is the stochastic trend (cumulative innovations). Assuming $y_0 = 0, \varepsilon_t \sim iid N(0, \sigma_w^2)$ it can be shown that:

$$E(y_t) = at$$

$$var(y_t) = t\sigma_w^2$$

Where t is the time horizon of the difference. Since its variance grows with time, this process is not stationary. Assuming $a = 0, \sigma_w = 1$:

$$E(y_t) = 0$$

$$var(y_t) = t$$

TEST FOR RANDOM WALK HYPOTHESIS (RWH)

In this section, two techniques will be introduced for testing of the Random Walk Hypothesis, namely, Variance Ratio Test and the Generalized Hurst Component.

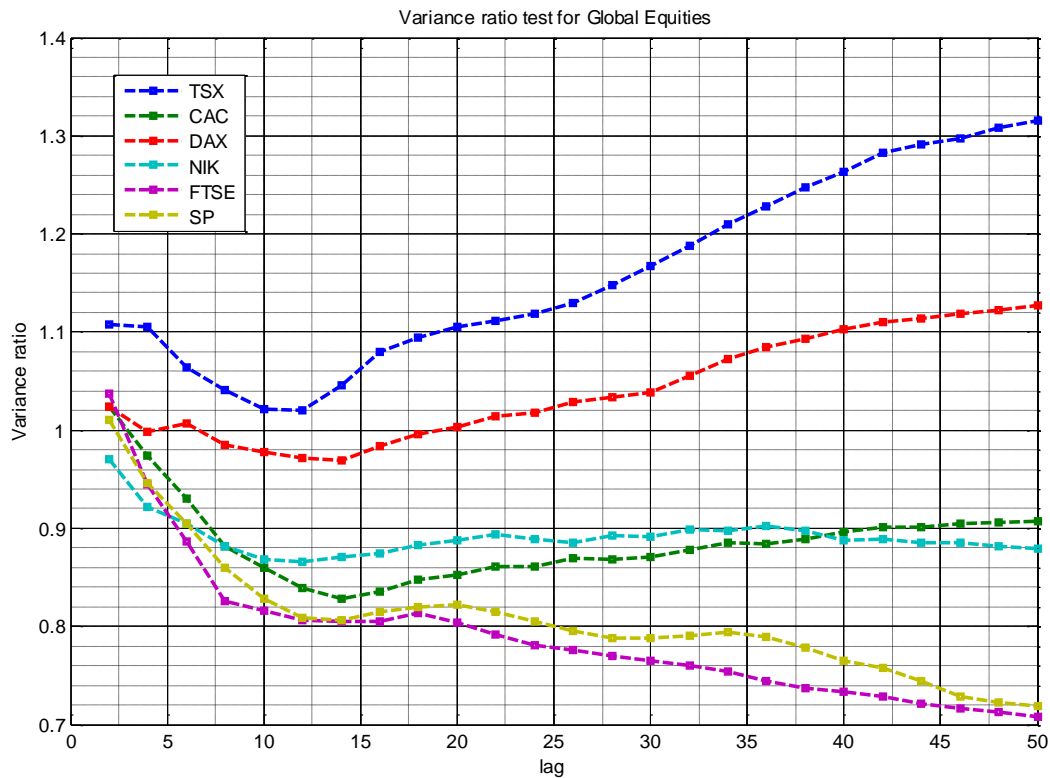
VARIANCE RATIO TEST

One technique that is useful in determining whether a process is random walk is called Variance Ratio Test. Intuitively speaking, if the log of the asset prices follow Geometric random walk, then its variance is expected to grow linearly with time, in which case the variance ratio should be equal to 1. For a mean reversion process, the price diffuses slower than a RWH, and its variance ratio should be less than 1. If a variance ratio is significantly

greater than one, then we cannot neglect the covariance in the short term, and it implies that the log returns exhibit positive serial correlations. Mathematically, based on model suggested by (Lo, 1988), RWH can be checked by comparing $var(Y_t - Y_{t-q})/q$ with $var(Y_t - Y_{t-1})$, where $Y_t = \log(P_t)$ and $var(Y_t - Y_{t-q})$ is the variance of q period log returns. The variance ratio (VR) is defined as:

$$VR_q = \frac{var(Y_t - Y_{t-q})/q}{var(Y_t - Y_{t-1})}$$

In the following section, I show the variance ratio test result for log price of global equities:



It can be seen clearly that most global equities tend to exhibit mean reversion between 2 to 10 lags, although some of them start to behave trending behaviors at larger lags. Most prominently, Canadian TSX exchange demonstrates trending behaviors at intermediate lags (approximately 30 days onward), which suggests candidacy for momentum strategies.

The most mean-reverting series seem to be FTSE 100 and S&P 500 indices, which exhibit decreasing variance at most lags. This is consistent with the expectations since S&P 500 is well known for its tendency to mean-revert, and FTSE 100 is often largely influenced by movement in S&P 500. However, the test failed to reject the hypothesis of random walk at 5% significance level for various assets and lags combinations. Noticeably, the test failed to reject random walk hypothesis for DAX at all lags tested.

GENERALIZED HURST EXPONENT

Another test that can be used together with variance ratio test is the Generalized Hurst Exponent (GHE). Mathematically, GHE can be expressed as:

$$\langle |Y_{t+\tau} - Y_t|^q \rangle \sim \tau^{qH(q)}$$

Where $H(q)$ is the Hurst exponent calculated from the scaling of the renormalized q -moments of the distribution. Usually $q = 2$ is the default setting for our testing. When $H = 0.5$, a process is assumed to be random walk; when $H < 0.5$, a process is expected to be mean-reverting; and when $H > 0.5$, a process is expected to be trending. Combined with the hypothesis testing results from variance ratio test, we can summarize our test statistics for random walk:

Symbol	Hurst	p-value	h	VR
Canadian TSX Composite	0.51	0	1	1.11
French CAC 40	0.47	0.21	0	1.02
German DAX	0.5	0.22	0	1.02
Japanese Nikkei 225	0.48	0.13	0	0.97
UK FTSE 100	0.46	0.06	0	1.04
US S&P 500	0.46	0.61	0	1.01

This table shows that TSX is trending with Hurst greater than 0.5 and variance ratio greater than one, and the hypothesis of random walk can be rejected with p-value equal to 0. S&P 500 and FTSE 100 exhibit the smallest Hurst exponents. However, the Variance ratio test cannot reject the null hypothesis of random walk at 5% significance level with p-value equal to 0.61 and 0.06, respectively.