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Time-Varying Channel Estimation using Pilot Symbol Assisted Modulation and Basis Expansion Model

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Abstract

In this paper, we address the problem of time-varying (TV) high-frequency (HF) channel estimation using Basis Expansion Model (BEM), adopting the pilot symbol assisted modulation (PSAT) approach. Conventional channel estimation techniques assume the TV channel coefficients to follow an uncorrelated stationary random process. Alternatively, the recently popularized basis expansion model attempts to capture the channel temporal variations by modeling the channel as superposition of finite sums of weighted kernels, thereby allowing the TV channel estimation problem to be reduced to the inference of fewer time-invariant (TI) basis coefficients. First, the various BEM model specifications are analyzed. Secondly, the BEM representation of the channel is formulated, and pilots aided channel estimation procedures using BEM are investigated under various scenarios. Next, we propose a semi-blind channel estimation algorithm relying on sensor array data, assuming a block transmission scheme. Finally, the BEM performance analysis and the proposed estimation algorithm are illustrated with simulations.

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1. Introduction

1.1. Background

Due to the existence of multipath propagation and delay spread, common wireless communication channels are frequency-selective. Due to the user mobility, the channel also often demonstrates time-selectivity. Such channels are thus called *doubly selective channel*. In such channels, the symbol transmission and reception may be severely impaired by phenomena such as channel fading, multipath-interference, delay spread, Doppler spread, etc.

To mitigate and compensate these negative channel distortions, equalizations are essential to be introduced on the receiver end. When the propagation channel is time invariant (TI), the channel response can be easily estimated and the efficient linear equalizers can thus be designed at low-complexity. However, when the channel is time-varying (TV), low complexity channel equalizer may be difficult to implemented without accurate knowledge of the TV propagation channel.

TV Channel estimation thereby demonstrates great significance in building effective wireless transmission systems. To perform the TV channel estimation, blind and semi-blind training based approaches have been proposed in the literature. When training based approach is adopted, known pilots need to be inserted periodically in each transmission block, in order to aid the channel estimation procedures. As pilots are included, the effective information rate of the channel will be reduced. Considering the band-limited applications, the valuable spectrum resources may be under-utilized owing to the required frequently and periodically inserted pilots. In this regard, blind channel estimation techniques that rely solely on the statistical properties of the received signals also attracted much attention. However, these blind approaches usually require long record of transmission data, and thus cast high computational load on the system. Therefore, for efficient and robust channel estimation, semi-blind and training based approaches are easier to implement and thus gained much popularity recently.

Due to the double selectivity of the TV channel, channel estimations need to adapt quickly in order to capture the rapid temporal variations of the channel. As far as training-based approach is concerned, a realistic modeling of such channel can involve large number of parameters to be estimated. Thus, it requires training symbols to be employed frequently, which greatly increase the computational load and reduce the transmission capacity.

For fast and robust implementation of TV channel estimation algorithm, a parsimonious modeling approach is called for. The Basis Expansion Model (BEM) has gained popularity recently, owing to its ease of implementation and reduced estimation complexity. BEM approach models the TV channel as a superposition of weighted kernels with distinct frequencies, thus convert the TV channel estimation problem into the estimation of much fewer kernel coefficients, which are assumed to be TI during each block duration. Several BEM types have

been proposed in the literature, including the Fourier analysis inspired complex-exponential basis and the Taylor expansion inspired polynomial and Legendre basis. In this paper, attention will be restricted on the semi-blind TV channel estimation algorithm design using the Basis Expansion Model.

1.2. Thesis Organization

In Chapter 2, the time-varying channel basics are briefly reviewed, including the concepts of channel fading and double-selectivity. In Chapter 3, the problem formulation of the research will be explored, which forms the main objective for the subsequent discussions. Chapter 4 introduces the representation of the system and signal model.

Traditional beamforming techniques are considered in Chapter 5, in which MMSE beamformer, also known as wiener filter, are discussed in details. This beamforming procedure is useful for rejecting interference and achieving desired spatial selectivity on the received signals. In our proposed channel estimation procedure, this technique is employed before the channel estimation step in order to decouple the multipath signals.

In Chapter 6, the Basis Expansion Model (BEM) is reviewed and investigated. Firstly, the BEM basics and BEM channel representations are covered. Followed by that, various BEM model types are discussed and formulated, including the recently developed complex-exponential, generalized complex-exponential and Legendre kernel models. Next, the time-varying (TV) channel fading gain and channel estimation procedure utilizing the basis expansion model will be explored and analyzed. Finally, a new BEM-based high frequency (HF) channel estimation procedure based on pilot-symbol-assisted-modulation (PSAM) and sensor array data is proposed.

In Chapter 7, the adaptive linear channel equalization techniques will be covered. To mitigate the negative channel distortion effect arising from the time and frequency selectivity of the channel, equalization procedure is essential to be included at the receiver end. The zero-forcing (ZF) and minimum mean square error (MMSE) equalizers will be explored and compared.

The simulation results will be presented in Chapter 8 to verify the effectiveness of the BEM inspired channel estimation techniques. As a proof of concept, various BEM channel specifications will be tested on simulated data. Next, the simulation of the proposed sensor array approach of TV channel estimation procedure based on basis expansion model will be demonstrated. Finally, discussions and analysis based on the simulation results will be included.

The thesis concludes on Chapter 9, with the future recommendations included.

2. Time-varying Channel

2.1. Propagation Channel

A channel is defined as a transmission path between a transmitter and receiver antenna. Pragmatically, the channel characteristics can be affected by multiple factors. For example, the existence of absorption, attenuation, dispersion, delay spread, Doppler spread, interference, motion and fading, etc¹, can cause the channel to behave in a complex manner.

2.1.1. Multipath Propagation Channel

The channel complexity will be further increased if multipath propagation occurs. Different from an ideal flat earth model, a typical multipath propagation channel cannot be characterized in a deterministic manner. There are various possible mechanism affecting the channel characteristics and causing multiple paths, such as scattering, refraction, reflection and diffraction. These multiple paths, which may differ in time delay and frequency offset, and also in phase and amplitude, will add up and make the superimposed signal stochastic in nature. Therefore, to represent these channels, statistical models must be employed.

2.2. Propagation Channel Characteristics

2.2.1. Channel Fading

In wireless communication systems, fading is referred as the deviation of the channel attenuation on the transmitted signal over the propagation channel. The fading process may be time and frequency selective in nature, and thus is often modeled as a stochastic process². Fading can be induced by either multipath propagation or shadowing. In a multipath induced fading, the replicas of signal in each path, with different amplitude, phase and angle of arrival, combine constructively and destructively, which cause the signal power to fluctuate over the propagation media. Besides, fading can be slow or fast. Fast fading is characterized by the rapid fluctuation of signal power. Such fading scheme can be best modeled by Rayleigh distribution when there is no direct line-of-sight. Alternatively, where there is a dominant path, a Rician distribution may produce less modeling error. On the other hand, slow fading is characterized by slow fluctuation in the mean signal power. This type of fading is often also referred as log-normal fading, since log-normal distribution tends to demonstrate good modeling accuracy.

2.2.2. Doppler Effect

The Doppler effect (a.k.a. Doppler shift), is a measure of the change in frequency components of a wave due to the relative motion between the observer and the source. Particularly, when the relative motion is approaching, the received frequency will be intensified; when the source recedes from the observer, the received frequency will be diminished.

As an illustration, when the relative motion between the source of wave and the observer is approaching, the sequential wave can be viewed as emitting from a source with relative closer

distance from the observer. Thus, comparatively, the transmitting wave will take less duration to reach the observer, causing a increase in the received frequency. Similarly, a decrease in the received frequency will be introduced when the source is receding from the stationary observer. For signals transmitting in a propagation channel, the total Doppler will be the combination of the Doppler spread induced by relative motion among source, observer and the propagation median.

In a multipath propagation channel, each multipath can have different Doppler spreads, due to the variations in the transmitting path, angle of arrival, scattering effects, etc. Mathematically, the Doppler spread can be measured as³:

$$D_k = \frac{v_{max}}{\lambda} \cos \alpha_k = f_{max} \cos \alpha_k$$

where D_k is the Doppler spread of the k th path, v_{max} is the maximum velocity of the moving object, λ is the wavelength of the propagation wave, α_k is the angle of arrival of the signal relative to the moving object, and f_{max} is the maximum Doppler frequency.

2.2.3. Doubly Selective Channel

Doubly selective channel denotes the type of channels with *time* and *frequency selectivity*⁴. With the ever growing demand for high frequency wireless communication applications, high data rate and high user mobility have become a new norm. The frequency selectivity is caused by high data rate and the presence of multipath fading. When the transmitting signal is going through channel effects, such as reflection, refraction and dispersion, etc, the resultant signals will superimpose constructively and destructively in a random manner. In this case, each transmitting symbol will spread over its adjacent symbols, which gives rise to inter-symbol interference (ISI). As result, the channel response shows frequency selectivity.

Another characteristic of doubly selective channel is the time-selectivity, which is caused by the mobility of the receiver or the transmitter. As discussed in the previous section, when the mobile user is in motion, Doppler effect will be introduced, and this gives rise to the time varying behavior of the propagation channel. In the paper, our focus will be the estimation and equalization of doubly selective channels.

3. Problem Formulation

In this chapter, we formulate the problem we want to address in this paper. In order to mitigate the negative effect of channel distortion, equalization is required at the receiver end. When the underlying channel is time-invariant(TI), standard equalization procedure can be applied to recover the transmitting symbols. On the other hand, when the channel is time-varying, the standard IT equalization approach fails to provide acceptable results. Conventional equalization procedures assume that the channel response is known at each processing moment, whose effectiveness relies on an efficient time-varying (TV) channel estimation procedure. Generally, two types of estimation methods are proposed in the literature. One approach relies on the stochastic statistic model, and another one, which is called the *basis expansion model (BEM)*, transforms the time-varying channel estimation problem to the estimation of time-invariant kernel coefficients. In the latter BEM approach, the TV channel response is expressed as a superposition of chosen basis (or kernel) functions with TI coefficients. In this thesis, we focus on the training-based BEM approach. We present two cases: In the first scenario, we evaluate the BEM modeling accuracy of fading gain estimation given complete channel statistics; In the second scenario, we evaluate the BEM modeling accuracy of fading gain and channel matrix estimation based on array data with training pilots.

3.1. Fading Gain Estimation with Known Channel Response

When the TV channel fading gain is known, the estimation problem is effectively reduced to a simple curve fitting problem, where the curve is fitted as a superposition of finite sums of kernel functions. This problem can be well solved using various BEM specifications, where the details of the solutions to this problem will be covered in Chapter 6.

3.2. Semi-blind Channel Estimation using Array Data

Having solved the first problem, we move on to a more parsimonious scenario. When the TV channel response is unknown, we seek a pilot based training method. Consider a block-transmission scheme, where fixed number of symbols are transmitted block-wise. In each block duration, we insert front-padded equal number of pilots, whose positions and contents are assumed to be known. Based on the training sequence, we estimate the noisy measurement of the channel. Having this noisy estimate, the problem is further reduced to the channel estimation of known channel statistics.

Using this procedure, we first perform the fading gain estimation. Subsequently, we move to use the array data, in which we attempt to estimate the entire channel matrix. To achieve this, we first simulate the transmitting data. Next, we perform *synchronization* to locate the frequency and time offset of the multi-path signals. Next, assuming the knowledge of the *array steering vectors*, we perform *MMSE beam-forming* to cancel the *inter-symbol interference (ISI)*. After beam-forming, we are able to recover the received noisy signal after channel fading effect for each propagation path. Next, we perform channel estimation using *Basis Expansion Model*.

4. The System Model

4.1. The Signal Model

Consider a time-varying channel with impulse response $\mathbf{h}(t; \tau)$ that corresponds to channel response at time t to a impulse response at time $t - \tau$, $\mathbf{s}(t)$ as a base-band continuous time signal to be transmitted with a symbol duration T_s , $\mathbf{x}(t)$ as a base-band continuous time signal to be received, $\mathbf{v}(t)$ as the zero-mean Gaussian white noise. Then, the noisy received signal $\mathbf{x}(t)$ is the convolution between the channel response and transmitting signal:

$$\mathbf{x}(t) = \int_0^{\infty} \mathbf{h}(t; \tau) \mathbf{s}(t - \tau) d\tau + \mathbf{v}(t)$$

Further, we apply Fourier Transform to the channel response $\mathbf{h}(t; \tau)$:

$$\mathbf{H}(f; \tau) = \int_0^{\infty} \mathbf{h}(t; \tau) e^{-j2\pi f t} dt$$

where $\mathbf{H}(f; \tau)$ is the Fourier representation of $\mathbf{h}(t; \tau)$. If $|\mathbf{H}(f; \tau)| = 0$ for frequency $|f| > f_d$, then f_d is regarded as the Doppler spread of the time-varying channel in frequency domain. If $|\mathbf{H}(f; \tau)| = 0$ for delay $|\tau| > \tau_d$, then τ_d is regarded as the delay-spread of the time-varying channel in time domain⁵.

If we sample the continuous-time signal model at sampling rate $f_s = 1/T_s$, then for discrete-time n , where n corresponds to the continuous time nT_s , the discrete-time baseband signal model can be represented as⁶:

$$\mathbf{y}[n] = \sum_{l=0}^L \mathbf{h}_l[n] \mathbf{s}[n - l] + \mathbf{v}[n], \quad n = 0, 1, \dots, N$$

where L is the length of the channel (maximum discrete time delay, which corresponds to LT_s in the time domain, given T_s is the actual sampling time), $\mathbf{h}_l[n]$ is the complex channel impulse response (or channel tape) at time n with delay l , and $\mathbf{h}_l[n]$ with $l > L$ is assumed to be zero. $\mathbf{s}[n - l]$ is the input signal at l th channel tape, $\mathbf{v}[n]$ is the AWGN noise with 0 mean and fixed variance σ_v^2 .

In this model, if $\mathbf{h}_l[n] \neq 0$ for $l \neq 0$, then Inter-symbol interference (ISI) will be introduced, which means that symbol at discrete time n will affect the transmission of the next $L - 1$ symbols. In this case, ISI cancellation techniques such as beam-forming and equalization need to be used, in order to mitigate this ISI effect.

Equivalently, the adopted transmission model can also be expressed in its matrix form as⁷:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}$$

where \mathbf{H} is the time-domain channel matrix.

4.2. Watterson Channel Model⁸

In this paper, we use Watterson Channel model to simulate the high frequency channel. Watterson Channel model is a stationary HF channel model proposed by Watterson et.al in 1970⁹. In general, HF channels are observed to be non-stationary in nature. However, for sufficiently small short duration (approximately 10 minutes) and for band-limited channels, Watterson will have good modeling performance.

Watterson model is implemented as a transversal adaptive filter, whose channel tapes are assumed to be time-varying and complex. As discussed before, the generic channel model can be described as:

$$\mathbf{y}[n] = \sum_{l=0}^L \mathbf{h}_l[n] \mathbf{s}[n-l] + \mathbf{v}[n], \quad n = 0, 1, \dots, N$$

where \mathbf{h}_l is the L -tap time-varying filter of a transversal structure, and L is the channel length. Specifically, the channel taps are generated through a Gaussian filtering process. To implement it, we filter a Gaussian white noise through a filter with Gaussian shape in the frequency domain, and incorporate the Doppler effect into the filter design, by setting the standard deviation of the filter power spectrum to half of the Doppler spread (W. N. Furman and J. W. Nieto):

$$|H_i(f)|^2 = \frac{e^{-2f^2/d_j^2}}{\sqrt{\frac{\pi d_j^2}{2}}}$$

where f is ranging from $-\infty$ to ∞ . If we apply the Inverse Fourier Transform on $H_i(f)$, then we arrive at the time domain filter taps:

$$f_j(t) = \sqrt{2}e^{-\pi^2 t^2 d_j^2}$$

where t is also ranging from $-\infty$ to ∞ .

4.3. PSAM in Block Transmission Schemes

We assume a block transmission scheme, in which equal number of symbols are transmitted block-wise. To aid the channel estimation procedure, we adopt the pilot symbol assisted modulation (PSAM). Technically, pilot symbols stand for known symbols at known positions upon transmission, which can be treated as training symbols. By having these pilot symbols employed, semi-blind channel estimation utilizing Basis Expansion Models can be performed.

Specifically, assume the total number of symbols to be transmitted in each block is N_b . We insert N_a leading pilots in discrete positions $\{1, 2, \dots, N_a\}$ in the first block, and information sequence in positions $\{N_a + 1, N_a + 2, \dots, N_b\}$. Subsequently, we repeat this treatments in each following transmission blocks. Eventually, all positions within index ranges $\{(k - 1) \times N_b + 1, (k - 1) \times N_b + 2, \dots, (k - 1) \times N_b + N_a\}$ will be filled with pilot symbols, and all positions within index ranges $\{(k - 1) \times N_b + N_a + 1, (k - 1) \times N_b + N_a + 2, \dots, k \times N_b\}$ will be filled with information sequence, where $k \in [1, N]$ indicates the block number to be transmitted, and N stands for the index of the last block.

5. Optimal Beamforming

5.1. Beamforming Basics

Beamforming is a signal processing technique widely used in sensor array processing domain for transmitting and receiving directional signals, in order to achieve desired spatial selectivity.

When sensor arrays are used for signal transmission and reception, it is useful if we can combine signals at particular angle-of-arrival constructively (spatial filtering) and cause other directional signals experience destructive interference (interference rejection and ISI cancellation)¹⁰. Assume the angles of arrival are fixed and \mathbf{w} is the weights we want to estimate, then the output after Beamforming can be expressed as:

$$\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t)$$

In the equation, $\mathbf{x}(t)$ denotes the input signal with the following structure:

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{a}_0 s(t) + [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_K] \begin{bmatrix} s_{i1}(t) \\ s_{i2}(t) \\ \vdots \\ s_{iK}(t) \end{bmatrix} + \mathbf{v}(t) \\ &= \mathbf{x}_s(t) + \mathbf{x}_i(t) + \mathbf{v}(t) \\ &= \mathbf{x}_s(t) + \mathbf{x}_u(t) \end{aligned}$$

where $s(t)$ is the desired transmitting information symbol, $s_{in}(t)$ is the n_{th} delayed transmitting symbol, $\mathbf{x}_s(t)$ is the desired sensor array signal with dimension N (N is the number of antenna arrays), $\mathbf{x}_i(t)$ is the undesired interference signal, $\mathbf{v}(n)$ is the zero mean constant variance Gaussian noise, $\mathbf{x}_u(t)$ is the combined unwanted signals that we wish to reject through Beamforming, and \mathbf{a}_i is the N-dimensional array steering vector for fixed angle of arrival θ_i

5.2. MMSE Beamforming and Wiener Filter

One effective design specification is the Minimum Mean Square Error (MMSE) criteria. Here, MMSE criteria refers to the estimation of the weights vector \mathbf{w} that minimizes the residual error power with respect to the reference signal $d[t]$. As a result, the cost function we want to minimize is:

$$\begin{aligned} J &= E[|\varepsilon(t)|^2] = E[|\mathbf{w}^H \mathbf{x}(t) - d(t)|^2] \\ &= E[(\mathbf{w}^H \mathbf{x}(t) - d(t))(\mathbf{w}^H \mathbf{x}(t) - d(t))^H] \end{aligned}$$

where $d(t)$ is the scalar reference signal, \mathbf{w} is the beam-former weights. For convenience of illustration, we ignore the time index and the cost function is further simplified as:

$$\begin{aligned}
J &= E[(\mathbf{w}^H \mathbf{x} - d)(\mathbf{w}^H \mathbf{x} - d)^H] = E[(\mathbf{w}^H \mathbf{x} - d)(\mathbf{x}^H \mathbf{w} - d^*)] = \\
&= E[\mathbf{w}^H \mathbf{x} \mathbf{x}^H \mathbf{w} - \mathbf{w}^H \mathbf{x} d^* - d \mathbf{x}^H \mathbf{w} + d d^*] \\
&= \mathbf{w}^H E[\mathbf{x} \mathbf{x}^H] \mathbf{w} - \mathbf{w}^H E[\mathbf{x} d^*] - E[d \mathbf{x}^H] \mathbf{w} + d d^* \\
&= \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} - \mathbf{w}^H \mathbf{r}_{xd} - \mathbf{r}_{xd}^H \mathbf{w} + d d^*
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{R}_{xx} &= E[\mathbf{x} \mathbf{x}^H] \\
\mathbf{r}_{xd} &= E[\mathbf{x} d^*]
\end{aligned}$$

To minimize the cost function, we take the first derivative with respect to \mathbf{w}^H and set the output to 0:

$$\begin{aligned}
\nabla_{\mathbf{w}^*} J &= 0 = \mathbf{R}_{xx} \mathbf{w} - \mathbf{r}_{xd} \\
\mathbf{R}_{xx} \mathbf{w} &= \mathbf{r}_{xd}
\end{aligned}$$

which yields:

$$\mathbf{w}_{MMSE} = \mathbf{R}_{xx}^{-1} \mathbf{r}_{xd}$$

This solution is commonly known as the *Wiener Filter*, which is optimal without the knowledge of the spatial signature \mathbf{h} . This procedure can be viewed as training based, and thus requires the information of transmitted signal, or reference signal.

If we further assume the reference signal $d(t)$ is known as our desired signal $s(t)$, and the desired signals are uncorrelated with the interference and noise, then we can further assert:

$$\begin{aligned}
\mathbf{w}_{MMSE} &= \mathbf{R}_{xx}^{-1} E[\mathbf{x} d^*] \\
&= \mathbf{R}_{xx}^{-1} E[(\mathbf{x}_s + \mathbf{x}_i + \mathbf{v}) d^*] \\
&= \mathbf{R}_{xx}^{-1} E[(\mathbf{a}_0 \mathbf{s} + \mathbf{x}_i + \mathbf{v}) \mathbf{s}^*] \\
&= \mathbf{R}_{xx}^{-1} \mathbf{a}_0 E[\mathbf{s} \mathbf{s}^*] = \mathbf{R}_{xx}^{-1} \mathbf{a}_0 \mathbf{S}
\end{aligned}$$

where

$$\mathbf{S} = E[\mathbf{s} \mathbf{s}^*]$$

6. Time-Varying Channel Estimation with Basis Expansion Model

6.1. Time-Varying Channel Estimation Basics

In the design of low complexity channel equalizer, temporal variations of the channel has become a major obstacle. For capturing this time-varying nature, methods have been proposed to model it as uncorrelated random process. In this modeling approach, the channel fading taps are assumed to follow Gaussian zero mean Rayleigh fading process when the line-of-sight propagation is absent, or Gaussian non-zero mean Rician fading process when the line-of-sight propagation is present¹¹. As an alternative, basis expansion model (BEM) has recently gained popularity in TV channel estimation applications, due to its reduced computational complexity. When BEM is employed, the TV channel is approximated by a finite sum of chosen basis functions, whose coefficients can assumed to be time-invariant.

The choice of the BEM basis (or kernels) has thus given rise to intense debate among academics, since different basis types will provide distinct performance in various communication environment. Inspired by discrete Fourier transformation, complex exponential BEM (CE-BEM) and its oversampled version (GCE-BEM) have been proposed. Polynomial and Legendre basis BEM have also been discussed in the literature, owing to the acceptable modeling accuracy of finite-term Taylor expansion. In this paper, we will restrict our attention on the Basis Expansion Model based channel estimation techniques.

6.2. Semi-blind Channel Estimation

To estimate the TV channel coefficient, training based or purely blind methods can be employed. When blind approach is considered, the channel estimation only assumes the knowledge of the received signal, and the estimation is performed solely relying on the statistical properties of the received signals. This type of estimation approach generally requires a long data sequence to achieve acceptable estimation performance, and thus entails substantial computational complexity. To reduce this systematic load, *Pilot Symbol Assisted Modulation* approach is exploited as an alternative. This type of approach, when designed properly, only requires a sufficiently small sequence of training symbols in each transmission block to achieve comparable performance, and hence is also called a *semi-blind* approach. This pilot assisted approach does not require length data record, and are therefore easier to implement in practical applications.

6.3. BEM Channel Representation

Basis Expansion Model (BEM) has recently evolved into a popular choice of estimation for rapidly-varying doubly selective channels. These channels often exhibit multipath propagation caused by a few dominating reflectors and kinematics of the mobile.¹²In modeling time varying channels induced by Doppler spread, we express the channel response as superposition of time-varying basis functions, with time invariant coefficients. For estimation purpose, we assume

Doppler spread is known and chosen basis are fixed. With this treatment, the time-varying channel estimation problem is reduced to the estimation of time invariant basis coefficients.

Several basis choices have been proposed in the literature, including the popular complex exponentials and polynomials. We use \mathbf{B}_q to represent the general basis functions at its q th order, the discrete-time finite-length channel taps $\mathbf{h}_l[n]$ can be expressed as:

$$\mathbf{h}_l[n] = \sum_{q=0}^Q h_{l,q} \mathbf{B}_q[n]$$

where $q = 0, 1, 2, \dots, Q$ are the BEM component order, Q is the maximum BEM order, $h_{l,q}$ is the basis coefficient of q th order and l th delay (in the sequel, $h_{l,q}$ and $h_q(l)$ will be used interchangeably). Increase Q generally reduces the modeling order under moderate SNR, but brings in additional model complexity.

This equation can be equivalently expressed in its matrix form:

$$\mathbf{H} = \mathbf{F}\mathbf{G}$$

with

$$\mathbf{H} = \begin{bmatrix} h(0; 0) & \dots & h(0; L) \\ \vdots & \ddots & \vdots \\ h(N-1; 0) & \dots & h(N-1; L) \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} f_0(0) & \dots & f_Q(0) \\ \vdots & \ddots & \vdots \\ f_0(N-1) & \dots & f_Q(N-1) \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} h_0(0) & \dots & h_Q(L) \\ \vdots & \ddots & \vdots \\ h_0(L) & \dots & h_Q(L) \end{bmatrix}$$

where \mathbf{H} is the time varying channel matrix to be estimated of dimension $N \times L$, \mathbf{F} is the matrix of basis or kernel functions of dimension $N \times Q$, and \mathbf{G} is a matrix of the basis coefficients of dimension $Q \times L$. To use BEM for channel estimation, the basis coefficient matrix \mathbf{G} needs to be estimated, either using blind approach or semi-blind pilot assisted method. By choose proper BEM order Q , such that $Q \times L < N$ is satisfied, we arrive at a over-determined linear system with number of unknown parameters less than the block size N ¹³. Since the system is over-determined, we can solve the system either using Gradient Descent method through iteration, or using Normal Equation, which satisfies the least square criteria.

Finally, combining with our general signal model derived above, we can formulate the BEM representation of the received signal $\mathbf{y}[n]$ as:

$$\mathbf{y}[n] = \sum_{l=0}^L \left(\sum_{q=0}^Q h_{l,q} B_q[n] \right) \mathbf{s}[n-l] + \mathbf{v}[n]$$

where L is the channel length, and $\mathbf{v}[n]$ is a random AWGN noise, which affects the systematic modeling error.

6.4. Basis Expansion Model Review

6.4.1. Complex-Exponential Basis Expansion Model (CE-BEM)

Due to the simplicity of implementation, Complex Exponential Basis Expansion Model (CE-BEM) has gained popularity and been intensively investigated recently for time-varying channel estimation applications. This model is inspired by the Fourier analysis, which seek to represent an arbitrary function as a finite sum of its harmonics. Recall our general representation of the discrete-time base-band signal model:

$$\mathbf{y}[n] = \sum_{l=0}^L \mathbf{h}_l[n] \mathbf{s}[n-l] + \mathbf{v}[n], \quad n = 0, 1, \dots, N$$

where $\mathbf{h}_l[n]$ is the discrete-time channel response. We assume a block transmission scheme, where N_b symbols are transmitted block-wise. We assume the windowed block is periodic with fundamental frequency $w_f := \frac{2\pi}{N_b}$, then according to discrete Fourier transformation, we can convert the function into its Fourier series representation of order N_b , or estimate the function with less ordered finite sums, where each term will have a harmonic frequency of multiple of w_f . Then, during the an arbitrary block interval $\{t_r, t_r + N_b T_s\}$, where T_s is the symbol interval, the channel impulse response can be represented by complex exponential basis expansion of order Q :

$$\mathbf{h}_l[n] = \sum_{q=-Q}^Q h_q(l) e^{jw_q n}$$

where $h_q(l)$ represents the basis coefficient of order q , corresponding to delay of l , Q is the basis expansion order, which is fixed for all the blocks, $L := \lfloor \tau_d / T_s \rfloor$, with τ_d equal to the maximum delay spread, $w_q := \frac{2\pi}{N_b} \left(q - \frac{Q+1}{2} \right)$, $q = 1, \dots, Q$, and $N_b T_s$ is the continuous time block interval.

As we can see, $h_q(l)$ is a unary function of l , which is assumed to be invariant over the block interval for a given order q , but can be time-varying in subsequent blocks. Finally, the CE-BEM representation of the discrete-time base-band signal model can be expressed as:

$$\mathbf{y}[n] = \sum_{l=0}^L \left(\sum_{q=-Q}^Q h_q(l) e^{j\omega_q n} \right) \mathbf{s}[n-l] + \mathbf{v}[n], \quad n = 0, 1, \dots, N$$

Another generalized version of complex-exponential BEM is proposed (GCE-BEM)¹⁴ to further improve the modeling performance of CE-BEM. This version corresponds to oversample the previously critically sampled Doppler spectrum, which is equivalent to take more samples within the Doppler range. The general expression of GCE-BEM can be expressed as:

$$\mathbf{y}[n] = \sum_{l=0}^L \left(\sum_{q=-Q}^Q h_q(l) e^{j\frac{2\pi}{KN_b} \left(q - \frac{Q+1}{2} \right) n} \right) \mathbf{s}[n-l] + \mathbf{v}[n], \quad n = 0, 1, \dots, N$$

where K is a parameter that controls the oversampling rate of the model. It has been reported that this oversampling technique can mitigate the modeling error of CE-BEM at the jump discontinuities of the windowed sequence.

6.4.2. Legendre Basis Expansion Model (L-BEM)

One notable shortcoming of complex exponential based Basis Expansion is the *Gibbs phenomenon*¹⁵, which induces an error lower bound for Fourier based BEMs. Gibbs phenomenon suggests that Fourier sums overshoot at a jump discontinuity of any piece-wise continuous functions, and this overshoot will not fade away as frequency increases, but converge to a fixed height. As a remedy for this phenomenon, Legendre polynomials are proposed to replace the Fourier taps. In this way, the time-varying channel within a block duration is now denoted as a superposition of Legendre polynomials with time-invariant coefficients. As suggested in the literature¹⁶, the Taylor expansion approach is adopted as the foundation for this model. Recall that the standard *Taylor expansion* formula for an arbitrary function that is infinitely differentiable at complex-valued (or real-valued) a has the following formulations:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

where $f^{(n)}(a)$ denotes the n_{th} derivative of $f(x)$ at the point a , and $n!$ is the factorial function of n . As a special case, when $a = 0$, the Taylor series is also called the *Maclaurin series*. It is a common practice for one to retain only finite terms of the infinite Taylor series sum to estimate an arbitrary function:

$$f(x) \cong \sum_{n=0}^N \frac{f^{(n)}(a)}{n!} (x-a)^n + R_N$$

where N is the Taylor series order, and R_N is remainder corresponding to the N th order Taylor approximation. This approximation will have acceptable modeling accuracy when the $f(x)$ is concentrated only on lower order terms, and the error term R_N will decrease with larger N . Similarly, we can express the discrete-time channel response as:

$$\mathbf{h}_l[n] = \sum_{k=0}^K a_k(l)(n - n_0)^k + R_K(l)$$

where $(n - n_0)^k$ is the *polynomial basis*, $a_k(l)$ is the kernel coefficient that is assumed to be fixed in each block duration, and $R_K(l)$ is the error term corresponding to $(K + 1)^{th}$ order approximation. Since $\mathbf{h}_l[n]$ is band-limited in time domain with finite delay spread, the $R_K(l)$ will be strictly decreasing with increasing K . As $K \rightarrow \infty$, we expect $|R_K(l)| \rightarrow 0$.

Those polynomial kernels $\{(n - n_0)^0, (n - n_0)^1, \dots, (n - n_0)^K\}$ are independent over $\{-1, 1\}$, however, they are not orthogonal. To make the basic functions orthogonal, we apply Gram-Schmidt procedure on the polynomial kernels with respect to the inner product:

$$\oint_{-1}^1 P_m(x) P_n(x) dx = \begin{cases} 2 & m = n \\ 0 & m \neq n \end{cases}$$

Then we arrive at the *Legendre polynomial*, which is orthogonal within $\{-1, 1\}$. In fact, Legendre polynomials are the solutions to the following *Legendre differential equation*¹⁷:

$$\frac{d}{dx} \left[(1 - x^2) \frac{d}{dx} P_n(x) \right] + n(n + 1)P_n(x) = 0$$

where $P_n(x)$ is the Legendre polynomial with order n , which can be solved using standard *power series method* and expressed using the *Rodriguez's formula*:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^{(n)}}{dx^n} [(x^2 - 1)^n]$$

It can be easily shown that $P_0(x) = 1$ and $P_1(x) = x$. Then, by adopting *Bonnet recursion*, the higher order Legendre polynomial can be expressed recursively as:

$$P_{n+1}(x) = \frac{(2n + 1)xP_n(x) - nP_{n-1}(x)}{n + 1}$$

The Legendre polynomials up to order 6 is plotted as follows:

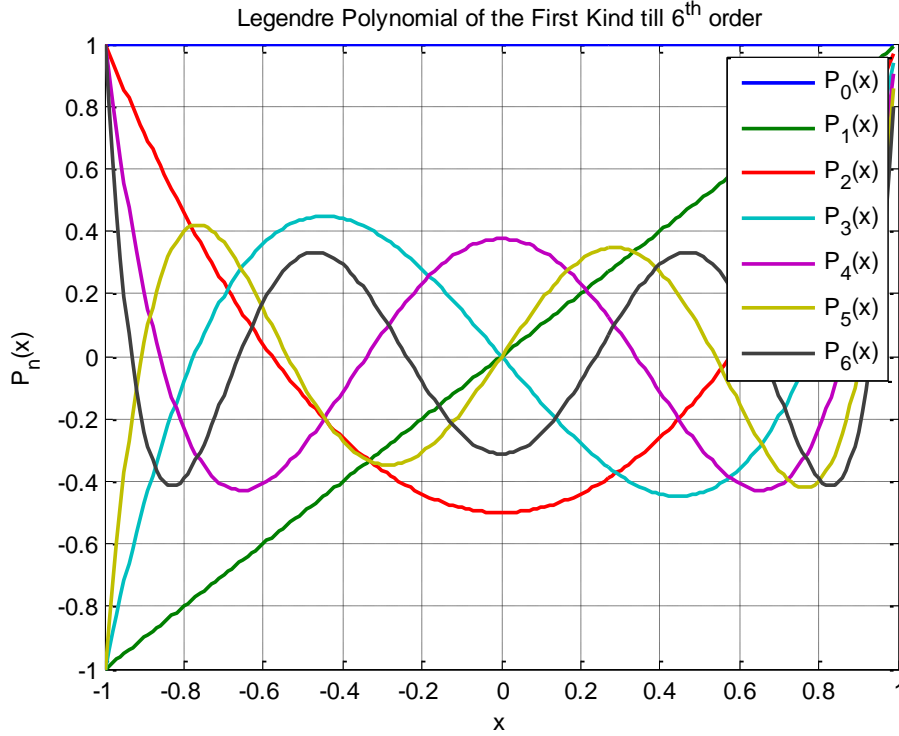


Figure 1. Legendre Polynomial of the first kind

Next, we express the channel as the generic BEM channel representation:

$$\mathbf{H} = \mathbf{F}\mathbf{G}$$

with

$$\mathbf{F} = \begin{bmatrix} 1|_{x=\frac{0}{N}} & x|_{x=\frac{0}{N}} & P_2(x)|_{x=\frac{0}{N}} & \cdots & P_Q(x)|_{x=\frac{0}{N}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1|_{x=\frac{N-1}{N}} & x|_{x=\frac{N-1}{N}} & P_2(x)|_{x=\frac{N-1}{N}} & \cdots & P_Q(x)|_{x=\frac{N-1}{N}} \end{bmatrix}$$

where $x = \{\frac{0}{N}, \dots, \frac{N-1}{N}\}$ is the normalized time. By this normalization, we force x to be within $\{-1, 1\}$, so all the Legendre kernels will be orthogonal.

6.5. Channel Estimation with Basis Expansion Model

In the following sections, we will discuss the fading gain and channel estimation using Basis Expansion Model. In each estimation problem, we start with the base case where the real channel response is assumed to be known. Next, we assume only the knowledge of the transmitted symbols, and perform the channel estimation utilizing the noisy measurement of the channel. Subsequently, we move on to a more parsimonious scenario where real TV channel information

is unknown but known training data sequence are employed in each block duration to assist the channel estimation. Finally, we conclude this section by proposing the estimation procedure when array data are used.

6.5.1. Channel Estimation with Known Channel Response

In this sub-section, we explore the channel estimation using Basis Expansion Model. We assume the complex channel response \mathbf{H} is known. Recall the basis expansion model representation of the channel \mathbf{H} :

$$\hat{\mathbf{H}} = \mathbf{F}\mathbf{G}$$

where $\hat{\mathbf{H}}$ is the estimated channel matrix. Therefore, for known real channel \mathbf{H} and fixed kernel matrix \mathbf{F} , the coefficient matrix \mathbf{G} should be chosen such that the following is minimized according to *least-square*:

$$\|\mathbf{H} - \mathbf{F}\mathbf{G}\|^2$$

Next, we derive the optimal solution of \mathbf{G} :

$$\begin{aligned} J &= \|\mathbf{H} - \mathbf{F}\mathbf{G}\|^2 \\ &= (\mathbf{H} - \mathbf{F}\mathbf{G})(\mathbf{H} - \mathbf{F}\mathbf{G})^H \\ &= \mathbf{H}\mathbf{H}^H - \mathbf{H}\mathbf{G}^H\mathbf{F}^H - \mathbf{F}\mathbf{G}\mathbf{H}^H + \mathbf{F}\mathbf{G}\mathbf{G}^H\mathbf{F}^H \end{aligned}$$

Next, solve $\nabla_{\mathbf{H}} J = 0$ gives rise to the MMSE solution:

$$\mathbf{H} = \mathbf{F}\mathbf{G}$$

which yields:

$$\hat{\mathbf{G}} = \mathbf{F}^\dagger \mathbf{H}$$

or equivalently:

$$\hat{\mathbf{G}} = (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H \mathbf{H}$$

where $\hat{\mathbf{G}}$ is the least-square estimation of time-invariant kernel matrix, and \mathbf{F}^\dagger denotes the *Moore–Penrose pseudo-inverse*. Hence, the least-square solution of \mathbf{H} can be obtained as:

$$\hat{\mathbf{H}} = \mathbf{F}\hat{\mathbf{G}}$$

6.5.2 Channel Estimation with Known Transmitted Symbols

When only the transmitted symbols are known, more modeling errors will be introduced, since the real channel response we used in *Section 6.5.1* needs to be estimated. In order to utilize the formulation in *Section 6.5.1*, one approach is to involve the noisy measurement of \mathbf{H} ¹⁸.

We define the signal model of the channel fading gain associated with a particular path or delay:

$$\mathbf{y}_l[n] = \mathbf{h}_l[n]\mathbf{s}[n] + \mathbf{v}[n], \quad n = 0, 1, \dots, N$$

where l is the discrete delay time. Therefore, ignoring the effect of the AWGN noise, we arrive at the scalar form of the noisy measurement of the channel for this particular delay:

$$\tilde{\mathbf{h}}_l[n] = \frac{\mathbf{y}_l[n]}{\mathbf{s}[n]} = \mathbf{h}_l[n] + \tilde{\mathbf{v}}[n]$$

where $\tilde{\mathbf{v}}[n] = \mathbf{v}[n]/\mathbf{s}[n]$. Repeat this procedure for fading gains associated with each delay offsets, we get the noisy measurement of the channel, \mathbf{H}_n . Given the noisy effect is acceptable, we reduce the estimation problem to the case with perfect channel knowledge. Similarly, the time-invariant kernel coefficients can be expressed as:

$$\hat{\mathbf{G}} = (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H \mathbf{H}_n$$

Hence, the least-square solution of \mathbf{H} can be obtained as:

$$\hat{\mathbf{H}} = \mathbf{F} \hat{\mathbf{G}}$$

6.5.3. Channel Estimation with Pilot Symbol Assisted Modulation

In practice, we will not know the complete transmission sequence, and only a series of pilot symbols at known positions will be known to the receiver. This constraint inspires us to use a semi-blind channel estimation approach. Specifically, we adopt the technique of *pilot symbol assisted modulation* (PASM)¹⁹.

In each block duration of N_B transmission symbols, we insert N_p pilots at known positions $\{n_1, n_2, \dots, n_p\}$. Therefore, the signal model for the fading gain at known pilot positions can be expressed as:

$$\mathbf{y}_{l,p}[n] = \mathbf{h}_{l,p}[n]\mathbf{s}_p[n] + \mathbf{v}_p[n], \quad n = n_1, n_2, \dots, n_p$$

where $\mathbf{y}_{l,p}$, $\mathbf{h}_{l,p}$, \mathbf{s}_p and \mathbf{v}_p denote the received signals, channel fading gain, transmitted symbols and Gaussian noise at known pilot positions. Hence we have the noisy measurements of the channel:

$$\widetilde{\mathbf{h}}_{l,p}[n] = \frac{\mathbf{y}_{l,p}[n]}{\mathbf{s}_p[n]}$$

Repeat this procedure for fading gains associated with each delay offsets, we get the noisy measurement of the channel, \mathbf{H}_{p_noisy} . And we can similarly obtain the kernel coefficient matrix:

$$\widehat{\mathbf{G}}_p = (\mathbf{F}_p^H \mathbf{F}_p)^{-1} \mathbf{F}_p^H \mathbf{H}_{p_noisy}$$

Where $\widehat{\mathbf{G}}_p$ is the TI kernel coefficient matrix based on the training samples, and \mathbf{F}_p is the basis matrix of dimension $N_p \times Q$:

$$\mathbf{F}_p = \begin{bmatrix} f_0(0) & \dots & f_Q(0) \\ \vdots & \ddots & \vdots \\ f_0(N_p - 1) & \dots & f_Q(N_p - 1) \end{bmatrix}$$

with Q equal to the order of the basis expansion model. Assuming the $\widehat{\mathbf{G}}_p$ is a proper approximation of the real windowed channel basis coefficients, the least-square solution of \mathbf{H} can be obtained as:

$$\widehat{\mathbf{H}} = \mathbf{F} \widehat{\mathbf{G}}_p$$

where \mathbf{F} is the basis matrix of dimension $N \times Q$. Recall that, given the window length N , each elements of \mathbf{F} are only dependent on the basis choice and the basis expansion model order Q .

6.6. Channel Estimation under Block Transmission using Sensor Array Data

In the sequel, we propose a realistic approach for estimating the multipath channel fading gain based on sensor array data. Similarly, we discuss the case when the full transmitted symbols are known and the case with only the pilot symbols employed. For the latter pilot assisted approach, we further explore it in a block transmission scheme, in which equal number of leading pilots are inserted in each transmission blocks.

6.6.1. Channel Estimation with Known Transmitted Symbols

In the proposed the sensor array approach, we assume that the angle of arrival, number of frequency offset and time delay for each multipath are known, and thus the steering vector for each path can be well estimated.

When M sensors are employed, we will have M observations for each sampling period, one obtained from each sensor. As discussed in *section 5.1.1*, considering the inter-symbol interference and added Gaussian noise, the received array signal can be expressed as.

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{a}_0 x(t) + [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_K] \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ \vdots \\ x_{iK}(t) \end{bmatrix} + \mathbf{v}(t) \\ &= \mathbf{y}_s(t) + \mathbf{y}_i(t) + \mathbf{v}(n) \end{aligned}$$

where $\mathbf{y}(t)$ is a $M \times 1$ vector, $x(t)$ is the transmitting signal after channel fading gain $\mathbf{h}(t)$ for a particular path, \mathbf{a}_0 is the known steering vector associated with $x(t)$. As one can see, $\mathbf{y}(t)$ can be decomposed as the symbol vector $\mathbf{y}_s(t)$, interference vector $\mathbf{y}_i(t)$, and the noise $\mathbf{v}(n)$. To recover $x(t)$ from the array signal $\mathbf{y}(t)$ and hence reject the inter-symbol interference, we perform MMSE beamforming:

$$x(t) = \mathbf{w}_{MMSE}^H \mathbf{y}(t)$$

where

$$\mathbf{w}_{MMSE} \cong \mathbf{R}_{xx}^{-1} \mathbf{a}_0 \mathbf{S}$$

with $\mathbf{R}_{xx} = E[\mathbf{y}\mathbf{y}^H]$ equal to covariance matrix of $\mathbf{y}(t)$ and $\mathbf{S} = E[ss^*]$ equal to the variance of s , assuming the interference and white Gaussian noise are both uncorrelated with the signal. We perform this step for each multipath for all the L paths:

$$\mathbf{w}_{l,MMSE} = \mathbf{R}_{xx}^{-1} \mathbf{a}_l \mathbf{S}$$

$$[x_1(t), \dots, x_L(t)] = [\mathbf{w}_{1,MMSE}, \dots, \mathbf{w}_{L,MMSE}]^T \mathbf{y}(t)$$

where \mathbf{a}_l , $\mathbf{w}_{l,MMSE}$, $x_l(t)$ are the array steering vector, MMSE optimal weights, and signal after channel distortion for the l_{th} path, respectively. Hence, we've decoupled the L multipath signals $\{x_1(t), \dots, x_L(t)\}$ from the superimposed signal $\mathbf{y}(t)$.

Next, we perform the channel fading gain estimation for each fading path. For notational simplicity, we use $\mathbf{x}(t)$ to denote a random fading path. Using the standard signal model, we can express $\mathbf{x}(t)$

$$x(t) = s(t) \times h(t) + n(t)$$

Then by incorporating the *Generalized Complex Exponential- Basis Expansion Model* (GCE-BEM) representation of the channel, the above equation can be expressed as:

$$x(t) = s(t) \times \left(\sum_1^Q g_q \times e^{jw_q t} \right) + n(t)$$

$$\begin{aligned}
&= s(t) \times \left([g_1 \quad \dots \quad g_Q] \begin{bmatrix} e^{jw_1 t} \\ \vdots \\ e^{jw_Q t} \end{bmatrix} \right) + n(t) \\
&= [g_1 \quad \dots \quad g_Q] \begin{bmatrix} s(t) \times e^{jw_1 t} \\ \vdots \\ s(t) \times e^{jw_Q t} \end{bmatrix} + n(t) \\
&= \mathbf{g}^T \times \mathbf{e}_t + n(t)
\end{aligned}$$

where Q is the BEM order, w_q is the basis frequency of order q , $\mathbf{g} = [g_1 \quad \dots \quad g_Q]^T$ is the kernel coefficients vector for GCE-BEM model, $\mathbf{e}_t = \begin{bmatrix} s(t) \times e^{jw_1 t} \\ \vdots \\ s(t) \times e^{jw_Q t} \end{bmatrix}$ is the kernel vector at time t , each multiplied by the transmitting signal $s(t)$, In discrete time format, the above equation can be modified as:

$$\begin{aligned}
x(n) &= s(n) \times \left([g_1 \quad \dots \quad g_Q] \begin{bmatrix} e^{jw_1 n} \\ \vdots \\ e^{jw_Q n} \end{bmatrix} \right) + v(n) \\
&= [g_1 \quad \dots \quad g_Q] \begin{bmatrix} s(n) \times e^{jw_1 n} \\ \vdots \\ s(n) \times e^{jw_Q n} \end{bmatrix} + v(n) \\
&= \mathbf{g}^T \times (s(n) \times \mathbf{f}_n) + v(n) \\
&= \mathbf{g}^T \times \mathbf{e}_n + v(n)
\end{aligned}$$

with $w_q = \frac{q}{KN}$, where $K > 1$ is the over-sampling rate for the GCE-BEM, N is the window size, and $\mathbf{f}_n = \begin{bmatrix} e^{jw_1 n} \\ \vdots \\ e^{jw_Q n} \end{bmatrix} = \begin{bmatrix} f_1(n) \\ \vdots \\ f_Q(n) \end{bmatrix}$ is the basis vector at discrete time index n . Subsequently, consider a windowed sequence with window size N , the matrix form of the above equation can be expressed as:

$$\begin{aligned}
&[x(n_1) \quad \dots \quad x(n_N)]^T = \\
&[\mathbf{e}_{n_1} \quad \dots \quad \mathbf{e}_{n_N}]^T \times [g_1 \quad \dots \quad g_Q]^T + [v(n_1) \quad \dots \quad v(n_N)]^T
\end{aligned}$$

where \mathbf{e}_n is the kernel vector multiplied by the information symbol $s(n)$ at discrete time index n of dimension $Q \times 1$, and g_q is the kernel coefficient associated with kernel function of order q . The above formulation can be easily reduced to a more compact form:

$$\mathbf{X} = \mathbf{E}\mathbf{g} + \mathbf{r}$$

where $\mathbf{X} = [x(n_1) \ \dots \ x(n_N)]^T$ has dimension $N \times 1$, $\mathbf{E} = [\mathbf{e}_{n_1} \ \dots \ \mathbf{e}_{n_N}]^T$ has dimension $N \times Q$, $\mathbf{g} = [g_1 \ \dots \ g_Q]^T$ has dimension $Q \times 1$, $\mathbf{r} = [v(n_1) \ \dots \ v(n_N)]^T$ has dimension $N \times 1$.

Assuming fixed BEM order Q , complete knowledge of $x(n)$ and $s(n)$, the matrix \mathbf{X} and \mathbf{E} can then be accurately computed. Subsequently, we can solve the kernel coefficient vector \mathbf{g} by minimizing the MMSE objective function:

$$\min_{\mathbf{g}} \|\mathbf{X} - \mathbf{E}\mathbf{g}\|$$

which yields the least-square solution:

$$\hat{\mathbf{g}} = (\mathbf{E}^H \mathbf{E})^{-1} \mathbf{E}^H \mathbf{X}$$

Recall that the BEM representation of the channel fading gain has the following matrix form:

$$\mathbf{H} = \mathbf{F}\mathbf{g}$$

where $\mathbf{F} = [\mathbf{f}_1 \ \dots \ \mathbf{f}_N]^T$ is the basis matrix of dimension $N \times Q$. Therefore, the estimated channel fading gain for a given fading path can be expressed as:

$$\hat{\mathbf{H}} = \mathbf{F}\hat{\mathbf{g}}$$

6.6.2. Channel Estimation with Pilot Symbol Assisted Modulation

In practice, not all transmitted symbols will be known. Therefore, a pilot assisted modulation approach is needed instead to perform the channel estimation. Again, assuming a windowed block of size N , where N_p known pilots are inserted at positions $\{n_1, \dots, n_p\}$. Then, similarly, the signal model with BEM representation can be expressed as follows in discrete time setting:

$$\begin{aligned} [x(n_1) \ \dots \ x(n_p)]^T = \\ [\mathbf{e}_{n_1} \ \dots \ \mathbf{e}_{n_p}]^T \times [g_1 \ \dots \ g_Q]^T + [v(n_1) \ \dots \ v(n_p)]^T \end{aligned}$$

or in a compact form:

$$\mathbf{X}_p = \mathbf{E}_p \mathbf{g} + \mathbf{r}_p$$

Then, the basis coefficients channel can be expressed as:

$$\hat{\mathbf{g}} = (\mathbf{E}_p^H \mathbf{E}_p)^{-1} \mathbf{E}_p^H \mathbf{X}_p$$

And the estimated channel can be formulated as:

$$\hat{\mathbf{H}} = \mathbf{F}\hat{\mathbf{g}} = [\mathbf{f}_1 \quad \cdots \quad \mathbf{f}_N]^T (\mathbf{E}_p^H \mathbf{E}_p)^{-1} \mathbf{E}_p^H \mathbf{X}_p$$

6.6.3. Channel Estimation under Block Transmission Scheme

Under block transmission, we assume equal number of leading pilots and information symbols are transmitted during each block duration. For each block, suppose N_p leading pilots are inserted first, followed by N_s information symbols. At the end of each transmission block, we perform the above mentioned BEM based TV channel estimation procedure, utilizing an expanding window. In other word, at the end of k_{th} block, pilot information obtained from block 1 to k will be utilized to generate the channel estimation. As a result, we will have $N_p \times k$ equally spaced received pilot sequence available for channel estimation:

$$\mathbf{X}_{p,k} = [\mathbf{x}_{p1}^T \quad \cdots \quad \mathbf{x}_{pk}^T]^T$$

where $\mathbf{X}_{p,k}$ is the collection of received signals of the training pilots till the k_{th} block and $\mathbf{x}_{pk} = \left[\mathbf{x} \left((N_p + N_s) \times (k - 1) + 1 \right) \quad \cdots \quad \mathbf{x} \left((N_p + N_s) \times (k - 1) + N_p \right) \right]^T$ is the k_{th} received pilot sequence obtained from the leading N_p symbols from the k_{th} transmission block. This pilot representation corresponds to stack all the pilots as a long column vector. After this formulation, we can similarly represent the signal model using the BEM representation:

$$\begin{aligned} & [\mathbf{x}_{p1}^T \quad \cdots \quad \mathbf{x}_{pk}^T]^T = \\ & [\mathbf{e}_1 \quad \cdots \quad \mathbf{e}_{kN_p}]^T \times [\mathbf{g}_1 \quad \cdots \quad \mathbf{g}_Q]^T + [v(1) \quad \cdots \quad v(kN_p)]^T \end{aligned}$$

or in a more compact form:

$$\mathbf{X}_{p,k} = \mathbf{E}_{p,k} \mathbf{g} + \mathbf{r}_{k,p}$$

where $\mathbf{E}_{p,k}$ is the basis matrix multiplied by the pilot signal collected till the k_{th} block and $\mathbf{r}_{k,p}$ is the mixed Gaussian white noise. As a comment, in practice, the noise may not be white, and a whitening process may be employed, given the knowledge of the noise covariance matrix $\mathbf{R}_{k,p}$.

Finally, the channel can be solved as:

$$\hat{\mathbf{H}} = [\mathbf{f}_1 \quad \cdots \quad \mathbf{f}_{kN}]^T (\mathbf{E}_{p,k}^H \mathbf{E}_{p,k})^{-1} \mathbf{E}_{p,k}^H \mathbf{X}_{p,k}$$

7. Channel Equalization

Equalization is one of the critical steps for the removal of inter-symbol interference (ISI) induced by channel distortion. As a result, equalization has become central to the performance enhancement in contemporary wireless communication systems, and has been under active research since 1960's. In the sequel, we will focus on the derivation of linear equalizers, whose goal is to roughly revert the channel selectivity and to mitigate the ISI caused by the propagation channel. Firstly, we will investigate the Zero Forcing Equalizer (ZF), followed by Minimum Mean Square Error Equalizer (MMSE). Other equalization methods such as Decision Feed Back Equalizer (DFE) and Fractionally Spaced Equalizer (FSE) will not be covered here.

7.1. Zero- Forcing (Z-Forcing) Channel Equalization

For the ease of representation, we consider only the finite length case here. Our goal will be to design a weighting vector that effectively mitigate the ISI effect and satisfies the Zero-forcing criteria.

According to the discrete signal model we discussed on section 4.1, each received symbol will be affect by the previous $L - 1$ symbols due to the presence of ISI. To design the equalizer taps, firstly, we represent the discrete finite-length equalizer taps we want to estimate as \mathbf{w} :

$$\mathbf{w}^T = [w_0 \ w_1 \ w_2 \ \cdots \ w_{N-1}]$$

where N is the order of the finite-length filter. Then, the scalar form of the equalizer output is z_n :

$$z_n = \sum_{k=0}^{N-1} w_k y[n - k] = \mathbf{w}^T \mathbf{y}[n]$$

where z_n is the recovered symbol (scalar) after the equalization stage, and $\mathbf{y}[n]$ is the original information sequence before equalization. Ideally, z_n should contain contributions from only the current symbol $y[n]$ and the AWGN components with sufficiently small variance²⁰.

In matrix form, we can express the matched filter output that we want to equalize as²¹:

$$\begin{aligned} & \begin{bmatrix} y[n] \\ y[n-1] \\ \vdots \\ y[n-(N-1)] \end{bmatrix} \\ &= \begin{bmatrix} h[0] & h[1] & \cdots & h[L] & 0 & \cdots & 0 \\ 0 & h[0] & h[1] & \cdots & h[L] & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h[0] & h[1] & \cdots & h[L-1] \end{bmatrix} \begin{bmatrix} x[n] \\ x[n-1] \\ \vdots \\ x[n-(N-1)] \end{bmatrix} \\ &+ \begin{bmatrix} v[n] \\ v[n-1] \\ \vdots \\ v[n-(N-1)] \end{bmatrix} \end{aligned}$$

or in a more compact form:

$$\mathbf{y}[n] = \mathbf{H}\mathbf{x}[n] + \mathbf{v}[n]$$

Using this formulation, we derive the output signal from the Zero-forcing filter, z_n :

$$z_n = \mathbf{w}^T \mathbf{y}[n] = \mathbf{w}^T \mathbf{H}\mathbf{x}[n] + \mathbf{w}^T \mathbf{v}[n]$$

In order to make z_n contain only contribution from the current symbol, we need to force the contributions from adjacent symbols zero, which corresponds to the inversion of the channel response. Therefore, our goal is to set:

$$\mathbf{w}^T \mathbf{H} = \boldsymbol{\delta}^T$$

where delay parameter $\boldsymbol{\delta} = [0 \ 0 \ \dots \ 0 \ 1 \ 0 \ \dots \ 0]^T$, and position of the element 1 depends on the number of samples of delay δ . Solve this equation using least square yields:

$$\mathbf{w}_{ZF} = (\mathbf{H}\mathbf{H}^T)^{-1} \mathbf{H}\boldsymbol{\delta}$$

where \mathbf{w}_{ZF} is the equalizer taps that satisfies the zero-forcing criteria. In summary, the zero-forcing criteria ensures that the ISI is minimized. However, this procedure may have the effect of magnify the additive white noise, which cause the Zero-forcing Equalizer's performance on low SNR environment to be mediocre. For instance, \mathbf{w}_{ZF} enhances the noise components when H has a spectral null (Cenk Toker, 2011).

7.2. Minimum Mean Square Error (MMSE) Channel Equalization

Minimum Mean Square Error (MMSE) equalizer is another type of linear equalizer, which is designed to minimize the variance of the residual difference between the transmitted and equalized signals. Due to this least square criteria, MMSE equalizer can effectively equalize the frequency- selective channel, and meanwhile suppress the AWGN noise. This addresses the fundamental weakness of ZF equalizer. When there is absence of noise, the ZF and MMSE equalizer become identical; when noise is presented, MMSE may fail to cancel the residual ISI completely, but will be able to suppress the noise effectively.

We focus only on the finite length situation here. Similar as before, the MMSE output is:

$$z_n = \mathbf{w}^T \mathbf{y}[n] = \mathbf{w}^T \mathbf{H}\mathbf{x}[n] + \mathbf{w}^T \mathbf{v}[n]$$

where z_n is the equalizer output, \mathbf{w} is the MMSE equalizer finite-length taps, $\mathbf{y}[n]$ is the received signal after channel response, \mathbf{H} is the channel as *Toeplitz* Matrix, $\mathbf{x}[n]$ is the transmitted signal, and $\mathbf{v}[n]$ is the added white noise.

Based on MMSE criteria, our objective function that we need to minimize is:

$$\begin{aligned}
J &= E[|x[n - \delta] - x[n]|^2] = E[|\boldsymbol{\delta}^T \mathbf{x}[n] - \mathbf{w}^T \mathbf{H} \mathbf{x}[n] - \mathbf{w}^T \mathbf{v}[n]|^2] \\
&= E[(\boldsymbol{\delta}^T \mathbf{x} - \mathbf{w}^T \mathbf{H} \mathbf{x} - \mathbf{w}^T \mathbf{v})(\boldsymbol{\delta}^T \mathbf{x} - \mathbf{w}^T \mathbf{H} \mathbf{x} - \mathbf{w}^T \mathbf{v})^H] \\
&= E[(\boldsymbol{\delta}^T - \mathbf{w}^T \mathbf{H}) \mathbf{x} - \mathbf{w}^T \mathbf{v})(\mathbf{x}^H (\boldsymbol{\delta} - \mathbf{H}^H \mathbf{w}^*) - \mathbf{v}^H \mathbf{w}^*)]
\end{aligned}$$

To further simplify the objective function, we leverage on the assumption that signal and noise are uncorrelated, which implies the cross correlation to be zero, $E[xv^*] = 0$. Hence,

$$\begin{aligned}
J &= E[(\boldsymbol{\delta}^T - \mathbf{w}^T \mathbf{H}) \mathbf{x} \mathbf{x}^H (\boldsymbol{\delta} - \mathbf{H}^H \mathbf{w}^*)] + E[\mathbf{w}^T \mathbf{v} \mathbf{v}^H \mathbf{w}^*] \\
&= (\boldsymbol{\delta}^T - \mathbf{w}^T \mathbf{H}) E[\mathbf{x} \mathbf{x}^H] (\boldsymbol{\delta} - \mathbf{H}^H \mathbf{w}^*) + \mathbf{w}^T E[\mathbf{v} \mathbf{v}^H] \mathbf{w}^* \\
&= (\boldsymbol{\delta}^T - \mathbf{w}^T \mathbf{H}) R_{xx} (\boldsymbol{\delta} - \mathbf{H}^H \mathbf{w}^*) + \mathbf{w}^T R_{vv} \mathbf{w}^* \\
&= (\boldsymbol{\delta}^T - \mathbf{w}^T \mathbf{H}) \sigma_x^2 \mathbf{I} (\boldsymbol{\delta} - \mathbf{H}^H \mathbf{w}^*) + \mathbf{w}^T \sigma_v^2 \mathbf{I} \mathbf{w}^* \\
&= \sigma_x^2 - \sigma_x^2 \boldsymbol{\delta}^T \mathbf{H}^H \mathbf{w}^* - \mathbf{w}^T \mathbf{H} \sigma_x^2 \boldsymbol{\delta} + \mathbf{w}^T \mathbf{H} \sigma_x^2 \mathbf{H}^H \mathbf{w}^* + \mathbf{w}^T \sigma_v^2 \mathbf{I} \mathbf{w}^* \\
&= \sigma_x^2 - \sigma_x^2 \boldsymbol{\delta}^T \mathbf{H}^H \mathbf{w}^* - \mathbf{w}^T \sigma_x^2 \mathbf{H} \boldsymbol{\delta} + \mathbf{w}^T (\sigma_x^2 \mathbf{H} \mathbf{H}^H + \sigma_v^2 \mathbf{I}) \mathbf{w}^* \\
&= \sigma_x^2 - \mathbf{F}^H \mathbf{w}^* - \mathbf{w}^T \mathbf{F} + \mathbf{w}^T \mathbf{R} \mathbf{w}^*
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{F} &= \sigma_x^2 \mathbf{H} \boldsymbol{\delta} \\
\mathbf{R} &= \sigma_x^2 \mathbf{H} \mathbf{H}^H + \sigma_v^2 \mathbf{I}
\end{aligned}$$

To arrive at the MMSE optimum solution to this over determined quadratic function of \mathbf{w} , we take first derivative with respect to w and set it to 0:

$$\nabla_{\mathbf{w}^*} J = -\mathbf{F}^H + \mathbf{w}^T \mathbf{R} = 0$$

Hence,

$$\mathbf{F}^H = \mathbf{w}^T \mathbf{R}$$

And

$$\mathbf{w}_{MMSE}^T = \mathbf{F}^H \mathbf{R}^{-1} = \sigma_x^2 \boldsymbol{\delta}^T \mathbf{H}^H (\sigma_x^2 \mathbf{H} \mathbf{H}^H + \sigma_v^2 \mathbf{I})^{-1}$$

8. Simulation Studies

8.1. Performance Analysis of Kernel-based Channel Fading Gain Estimation

In this section, we list some popular BEM specifications and analyze their performance in channel estimation simulations. To access the modeling accuracy, we adopt *pilots aided training method*, where known training pilots are to be introduced in equally spaced known positions. Finally, a model comparison analysis concludes the discussion. This section will thus be viewed as a *proof-of-concept* analysis.

For ease of illustration, we simulate a sequence of 2000 random QPSK signals, with over sampling rate set to 2, and symbol transmission rate as 2400. To simulate the TV channel, we set the channel Doppler Spread equal to 1, in order to capture the temporal variations of the time-varying channel responses. For modeling the high-frequency (HF) channel, we adopt the popular *Watterson Channel Model*, which is stationary but proven to be valid for band-limited channels with sufficiently small durations (C. C. Watterson, 1970). We assume a sub-optimal pilots arrangement scheme, where equal number of pilots are inserted periodically, with pilots-symbol ratio equal to 1 to 10. This corresponds to 400 pilots in this simulation study. In addition, the signal-to-noise ratio (SNR) is set to $20dB$.

8.1.1. Performance Analysis of CE-BEM Model

In this sub-section, we analyze the channel estimation performance of the Complex-Exponential Basis Expansion Model (CE-BEM). We employ a CE-BEM with kernel order 6, which is an arbitrary choice given proper model complexity. It can be observed that the significant portion of modeling error occurs around the discontinuities of the windowed sequence, which is a major weakness of CE-BEM model. This shortcomings can be mitigated by introducing a proper chosen *over-sampling-rate* (OSR) and further reduced by replacing the Fourier taps with Legendre polynomials.

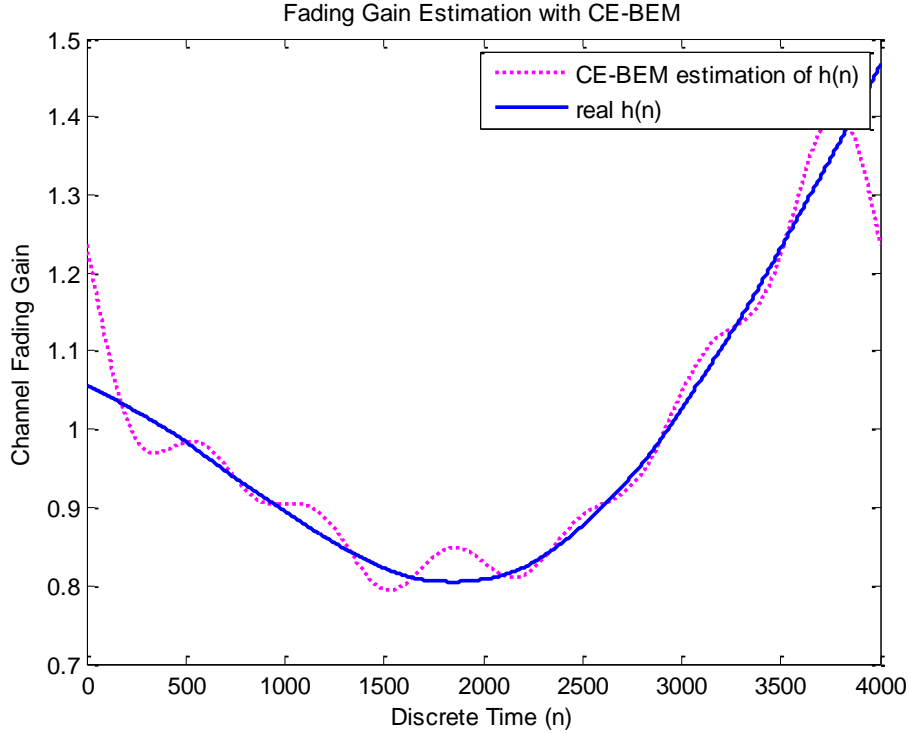


Figure 2. Fading Estimation with CE-BEM Model

8.1.2. Performance Analysis of GCE-BEM Model

In this sub-section, we analyze the channel estimation performance of the Generalized Complex-Exponential Basis Expansion Model (CE-BEM). We employ a model with kernel order equal to 10, and OSR K is set to 2 and 3 for comparison purpose. As illustrated below, by introducing over-sampling parameter $K > 1$, the modeling error of complex-exponential based BEM is substantially reduced. This version of CE-BEM is called GCE-BEM, and will be reduced to the base case of CE-BEM when we set $K = 1$. This added over-sampling parameter effectively addressed the modeling error concentrated in the edge of the windowed sequence of the CE-BEM case. However, it can be shown that further increase K cannot significantly boost the modeling accuracy.

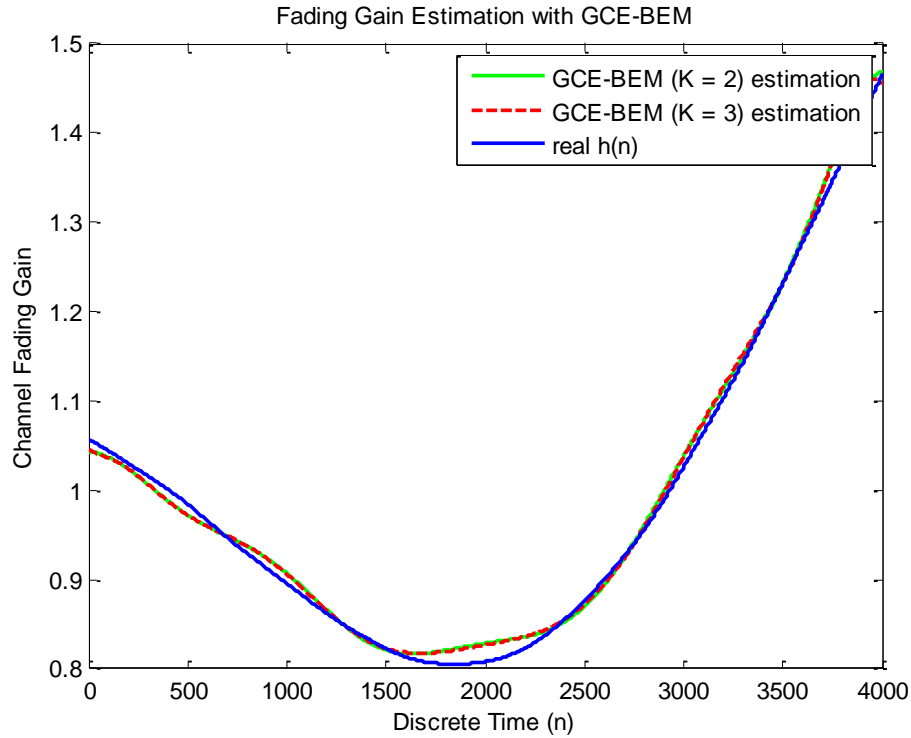


Figure 3. Fading Gain Estimation with GCE-BEM

8.1.3. Performance Analysis of Legendre-BEM Model

In this sub-section, we analyze the channel estimation performance of the Legendre Basis Expansion Model (CE-BEM). We use Legendre polynomial up to order 6 to approximate the TV channel. These Legendre basis, which is inspired by Taylor expansion and orthogonal decomposition, provide acceptable modeling accuracy, which is proved to be superior than the complex-exponential basis in some literatures. Most importantly, this BEM choice effectively addressed the fundamental weakness of Fourier based BEMs- the modeling error due to Gibbs phenomenon.

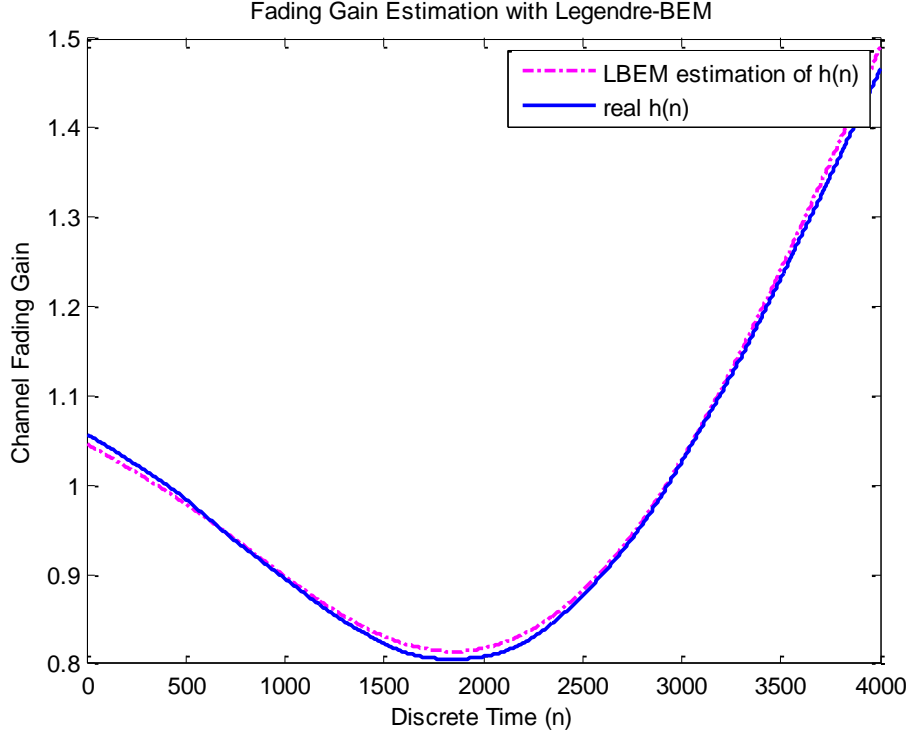


Figure 4. Fading Gain Estimation with Legendre BEM

8.2. High-Frequency Multipath Channel Estimation with an Array Approach

In this section, we discuss a more parsimonious simulation, where real *HF data structures* are used and an array approach is employed. As an outline, we first simulate the 8PSK transmitting signal with a *raised-cosine* pulse shaping. Next, we simulate the HF channel with the stationary *Watterson channel*, which is widely recognized in the HF community. After the preparation stage, we perform MMSE *beamforming* on the received array data, which are superimposed by multipath signals and added with zero-mean Gaussian white noise. After beam-forming stage, we perform training-based multipath *channel fading gain estimation*, adopting an approach inspired by the Basis Expansion Model (BEM). To evaluate the performance evolvement in a block-transmission scheme, we perform the above beamforming and channel estimation procedures in the end of each block duration. Specifically, we use an expanding window, where we include all the previous known block data in each subsequent channel estimations. Finally, we analyze the performance and comments on the results obtained.

8.2.1. Block Transmission Scheme and Pilots Arrangements

In the following discussions, we assume a block transmission scheme, where equal number of symbols and training pilots are inserted in each block duration. In each block, we transmit $N_p = 16$ pilot symbols first and $N_s = 32$ information symbols subsequently. Equivalently, we divide each block into two sub-blocks, where the first sub-block are the training sequence, P_t ,

and the second portion of the block is the sub-block of information sequence, P_s . In this simulation study, we transmit $N_b = 48$ symbols per block, and 20 blocks to be transmitted in total. To evaluate the channel estimation accuracy, we adopt an expanding window. Specifically, when performing channel estimation for the k_{th} block, we include all the known information from the previous k blocks. As a result, $k \times N_p$ equally spaced pilots are involved in the channel estimation. While the number of blocks transmitted increase, we expect our estimation to be more towards the real distribution of the channel response.

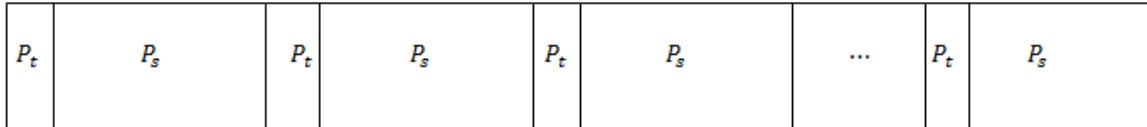


Figure 5. Pilots Arrangement in Block Transmission Schemes

8.2.2. Transmission Data

To start with, we simulate the transmission data, which in this case is assumed to be random 8PSK symbols with a *raised-cosine* pulse shaping. This randomized symbols can be simulated by performing convolution between the 8PSK symbols and raised-cosine filters.

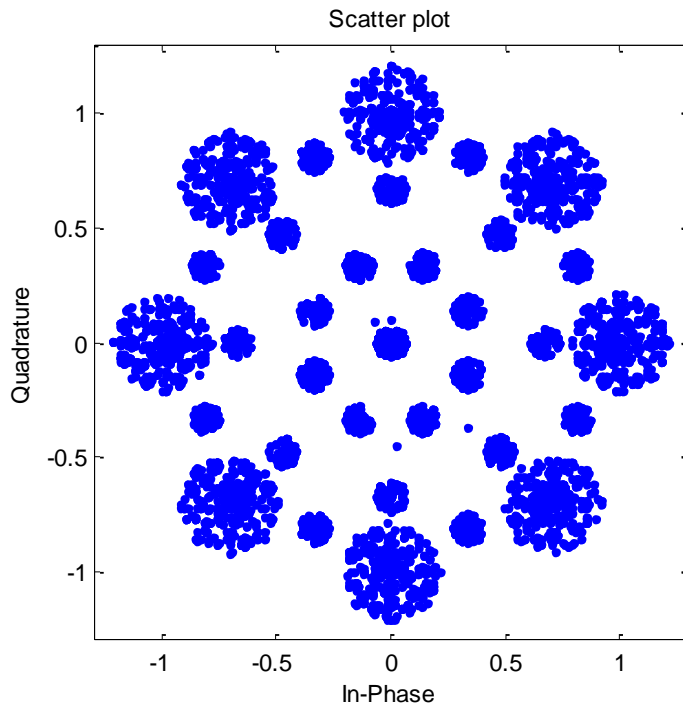


Figure 6. Transmitting 8PSK Symbol With Raised-Cosine Pulse-Shaping

8.2.3. Channel Generation

To simulate the HF channel, we adopt the Watterson Channel model, where the channel fading process is effectively implemented as a truncated Gaussian filter. We simulate the channel with 3 unknown multipath with specified path signal power, noisy power, path delay time and Doppler spread. As a result, we produce three random frequency and time selective channel fading gain, which we need to estimate in the sequel.

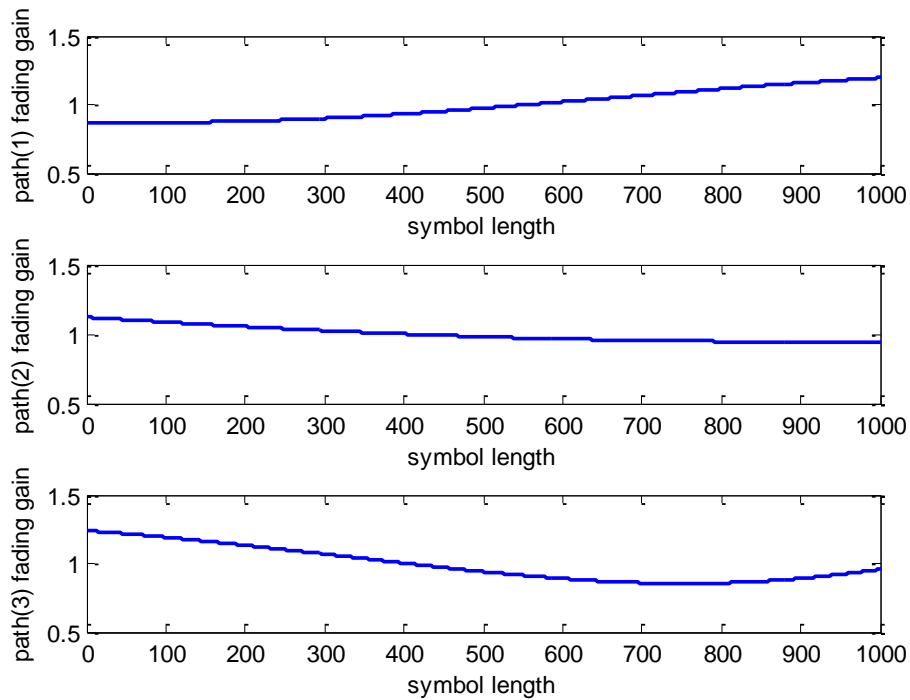


Figure 7. High-Frequency Channel Generated By Watterson Channel Model

8.2.4. MMSE Beamforming

From a realistic point of view, a sensor array may be used to collect the received multipath data. We use a array antenna of 4 equally spaced antenna elements. Hence, for a windowed sequence of block size N , we will collect array data of dimension $4 \times N$, N for each array. Assuming the knowledge of array steering vectors for each path, which can be estimated through synchronization in practice, we perform MMSE beamforming to each multipath. This procedure, although may introduce systematic inaccuracy, will effectively cancel out the inter-symbol interference between each delayed samples. After this procedure, we obtained a noisy measure of the each multipath.

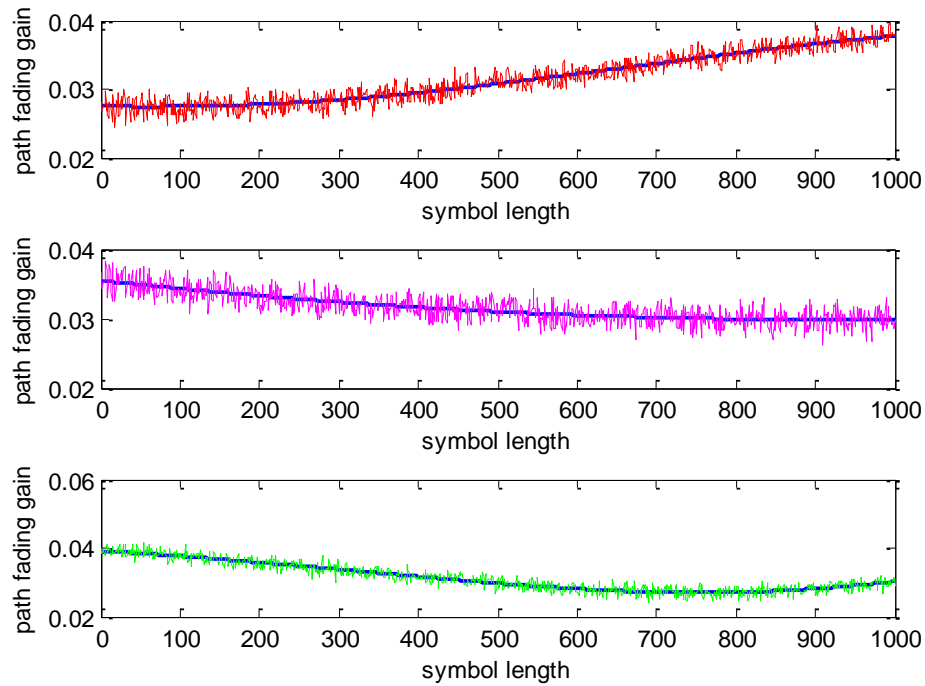


Figure 8. Decoupled Multi-Path Signal After MMSE Beamforming

8.2.5. BEM Channel Estimation

Finally, we perform the training-based BEM channel estimation procedures that introduced above. For illustration purpose, GCE-BEM is chosen as the selected BEM type, although other BEM types may also work well. We use the block transmission specification as discussed in 8.2.1, and repeat the estimation procedure at the end of each block-duration, with an expanding window. The maximum number of blocks transmitted are 20, which is selected arbitrarily. The BEM estimation and the real channel are plotted together below for ease of comparison:

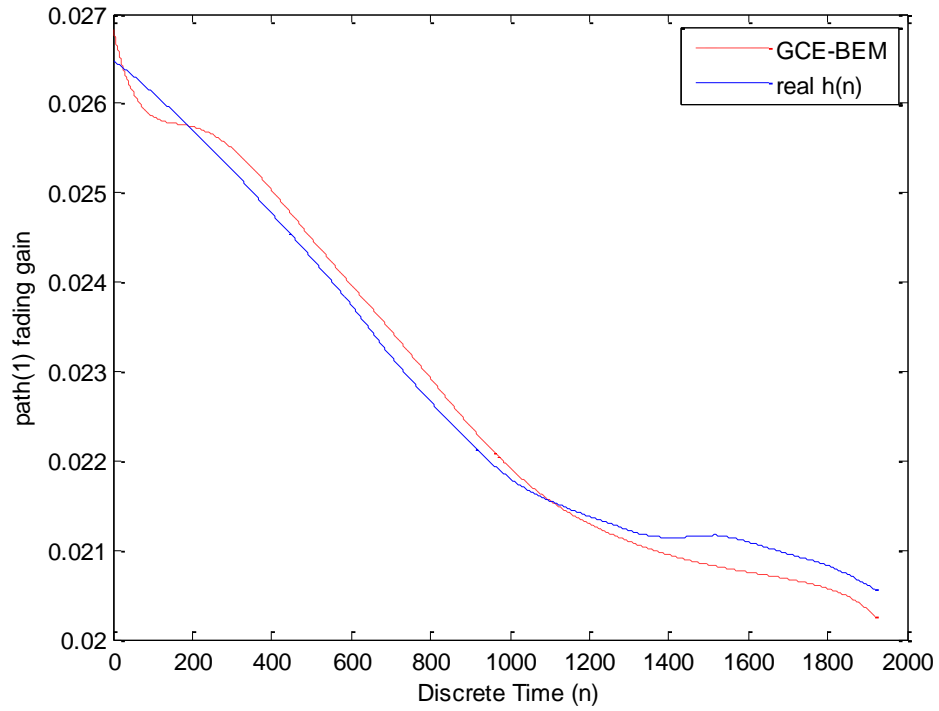


Figure 9. GCE-BEM Estimation of the First Multi-path

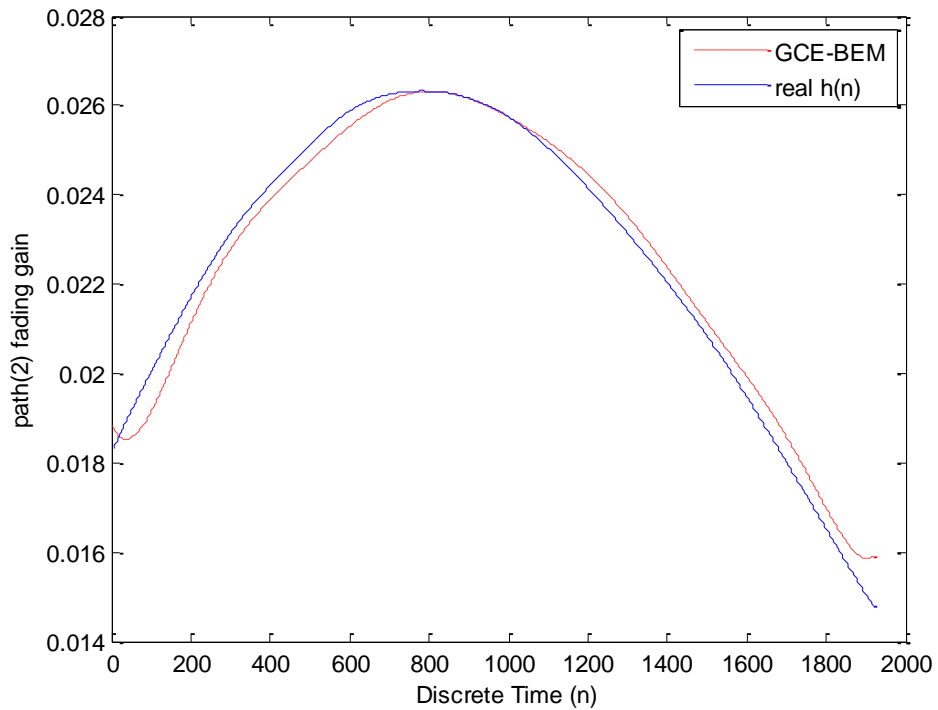


Figure 10. GCE-BEM Estimation of the Second Multi-path

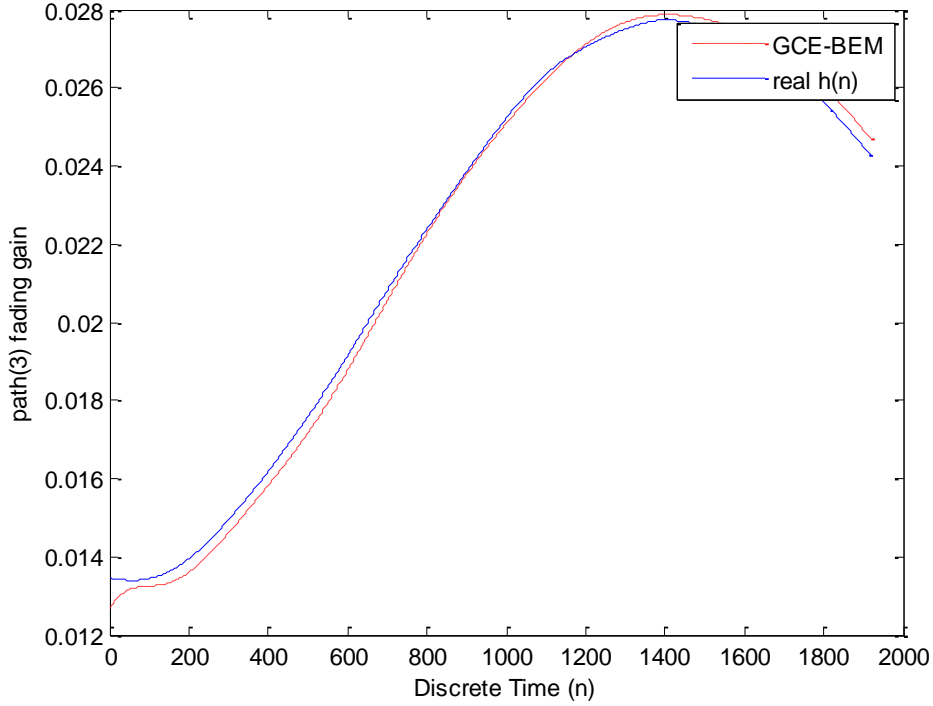


Figure 11. GCE-BEM Estimation of the Third Multi-path

8.3. Discussion and Analysis

8.3.1. Kernel Order Sensitivity

Consider Fourier Analysis inspired Complex-Exponential Basis Expansion Model (CE-BEM), the kernel matrix and kernel coefficient matrix are both functions of the kernel Q :

$$\mathbf{h}_i[n] = \sum_{q=-Q}^Q h_q(l) e^{jw_q n}$$

In theory, when Q increase, the Fourier harmonic expansion based estimation of a periodic function will approach its real values. Therefore, it's worthwhile to analyze the modeling performance as a function of kernel order Q , subject to $\frac{Q}{NT} \geq f_{max} = \max |f|$, where f_{max} is the maximum Doppler shift, and T is the sampling interval. Particularly, we analyze the case when a) there is no AWGN noise in presence; b) when the SNR is relatively high; c) when the SNR is low.

a) When there is no noise, the modeling performance will be in line with the theory, where more kernel functions will refine the estimation:

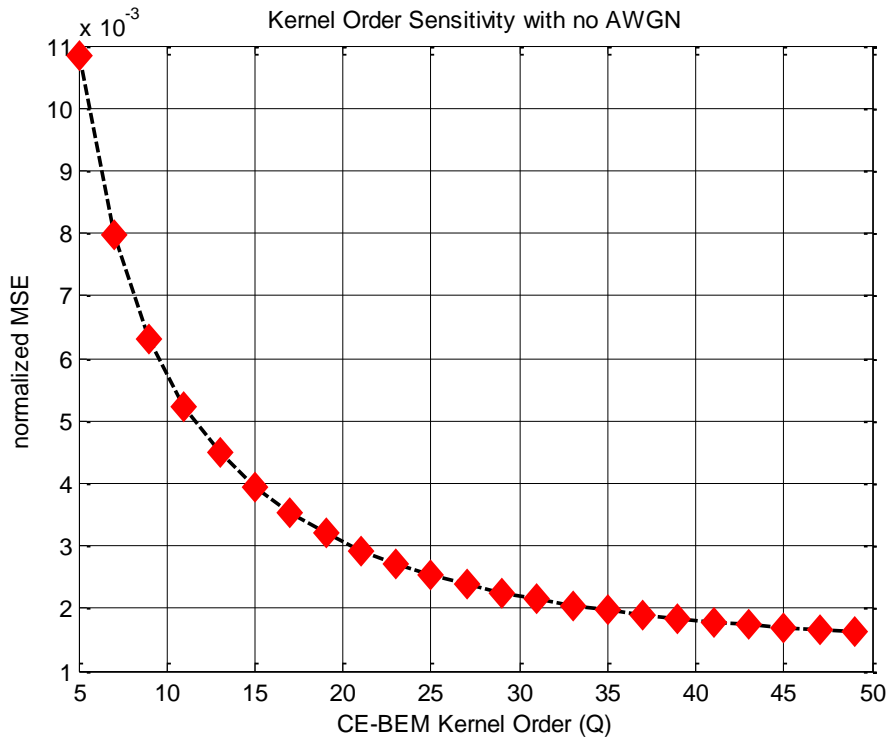


Figure 12. Kernel Order Sensitivity Analysis with No AWGN Noise

However, an error floor will remain as the order increases to infinity, which can be justified by the Gibbs Phenomenon in Fourier analysis.

b) When the SNR is moderate, the modeling accuracy will increase first but saturate and decrease at higher order, due to the induced AWGN interference:



Figure 13. Kernel Order Sensitivity Analysis with Moderate SNR

c) When the SNR is low, increasing the BEM order tends to magnify the AWGN interference in general:

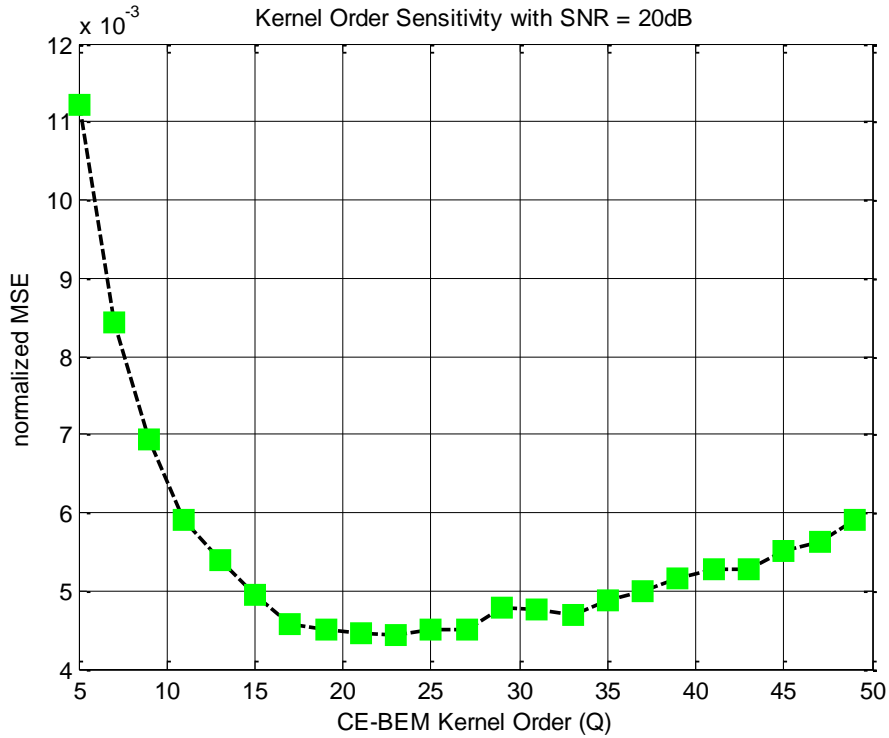


Figure 14. Kernel Order Sensitivity Analysis with Low SNR

8.3.2. Impact Analysis of Pilots Arrangement

Structures of pilots design can have a great impact on the channel estimation performance. In theory, pilot concentration, arrangement and power allocation can all affect the pilot guiding capability. In this analysis, we restrict our analysis on the effect of pilot concentration and arrangement. It is observed that, regardless of the pilot arrangement, the estimation performance will increase with rising number of pilots. When the number of pilots are fixed, the performance will be a function of the pilot arrangement. It is reported in the literature²² that periodic insertion of equally spaced zero-guarded pilot symbols was optimal. In comparison, we also adopt the technique when fixed number of leading pilots are adopted in each block. It is intuitive that, in the block-transmission scheme we proposed, the pilots arrangement will be effectively equivalent to the equally-spaced scenario, when longer block histories are used in the channel estimation of the recent block. By simulation, it is observed that the normalized MSE will be a decreasing function as the pilot concentration, and will be substantially lower when equally spaced pilots are employed.

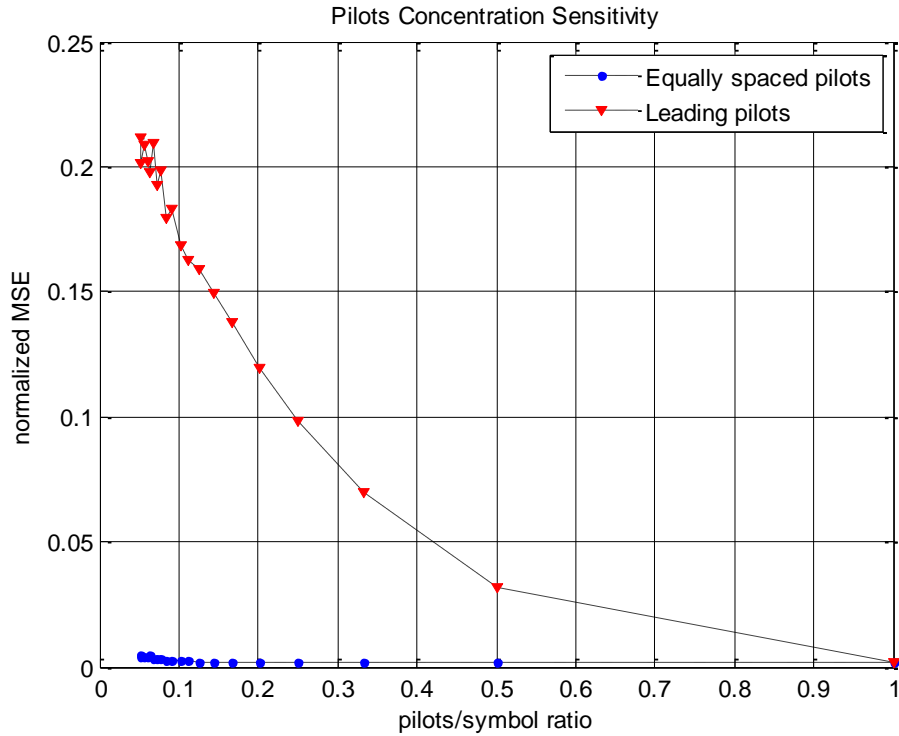


Figure 15. Pilots Concentration and Arrangement Analysis

8.3.3. SNR Sensitivity for Various BEM Kernels

In this sub-section, we move further to compare the modeling performance of various BEM choices in varying communication environments. First, we plot the normalized mean square error (MSE) versus the SNR. It can be shown that CE-BEM demonstrate substantially higher normalized MSE as compared to other BEM models in various SNR conditions, which may due to the increased modeling error in the discontinuities of the windowed sequence induced by Gibbs phenomenon. On the other hand, GCE-BEM with OSR equal to 3 offers substantial performance gain, but is still sub-optimal as compared with the L- BEM. When we further increase the SNR, the performance gain of L- BEM and GCE-BEM saturate, and their performances become comparable.

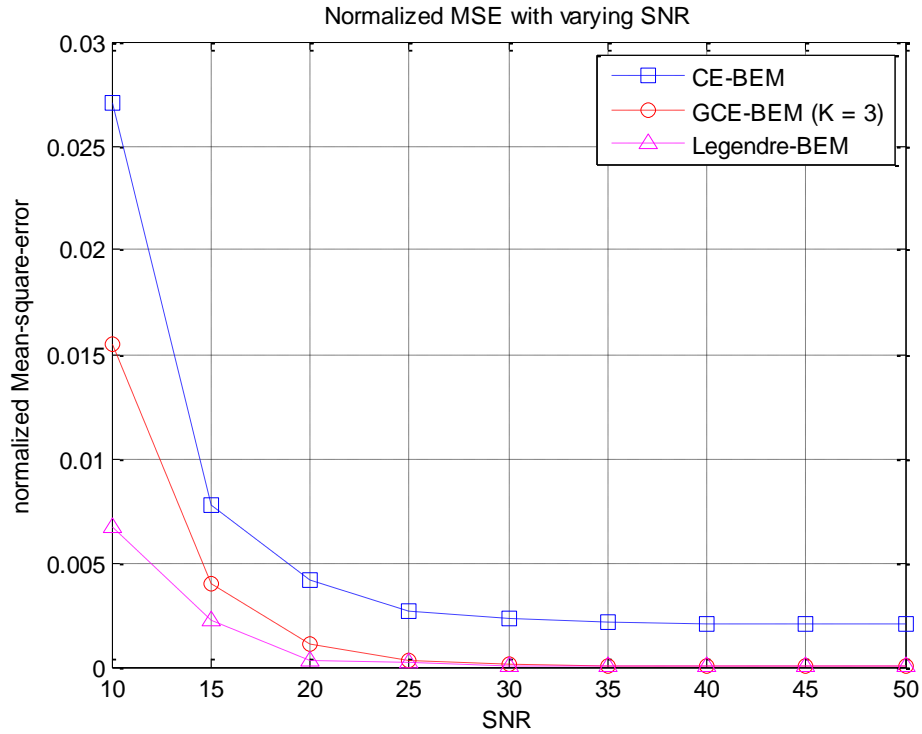


Figure 16. SNR Sensitivity for Various BEM Kernels

8.3.4. Modeling Error Analysis of the Fourier Expansions

Due to the simplicity of implementation, complex-exponential BEM (CE-BEM) is popularized recently. However, simulation studies have realized large modeling error of this BEM type, when oversampling of the Doppler spectrum is not used. This modeling error may due to the Gibbs phenomenon, which explains the overshoot of the Fourier series estimations at the jump discontinuities of a piece-wise continuous periodic function²³. When TV channel estimation is performed over a windowed block duration, discontinuities are observed at both ends of the window. Therefore, when CE-BEM is applied to perform the estimation, overshoot will be caused due to the Gibbs phenomenon, and modeling errors are thus introduced. To demonstrate this phenomenon, we simulate the Fourier series estimation of a periodic square wave:

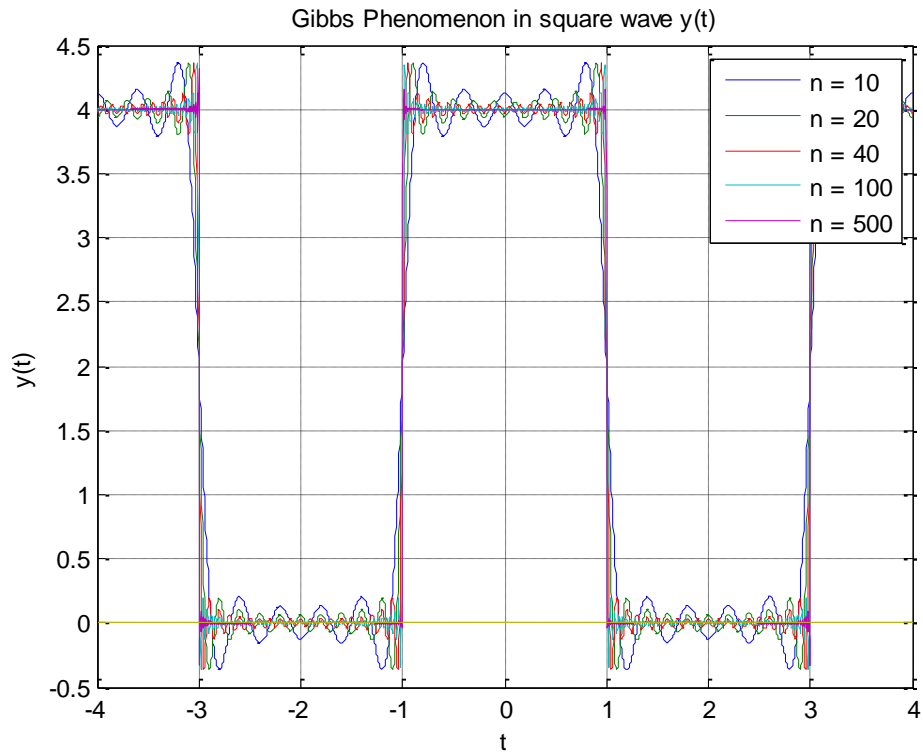


Figure 17. Gibbs Phenomenon Demonstration using a Square Wave

It can be viewed clearly that the overshoot on the edge of the jump does not die out with increasing Fourier terms, although the width and energy of the mismatch is converging to a finite limit.

9. Conclusion and Future Work

In this paper, we address the problem of time-varying (TV) high-frequency (HF) channel estimation using Basis Expansion Model (BEM), adopting the pilot symbol assisted modulation (PSAT) approach. We proposed a parsimonious pilot-assisted channel estimation procedure based on the sensor array data. Simulation studies show that it can obtain reasonable performance in a block transmission scheme with an expanding or sliding observation window.

In future, this research can be extended to cover the following aspects: firstly, research can be done to analyze the case when the array steering vector is unknown to the sensors; secondly, research can be extended to analyze the effect of different kernel choices; thirdly, one can combine this channel estimation procedure to the equalization stage, so to obtain more efficient equalizer for TV channels; finally, blind channel estimation algorithms can also be analyzed, given the goal is to achieve higher information and utilization rate of the channel.

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